Dipolar Bose-Einstein Condensates in Weak Anisotropic Disorder Potentials

Branko Nikolić[1,2], Antun Balaž[2], and Axel Pelster[3,4]

[1]Department of Physics, Freie Universität Berlin, Fabeckstraße 60, 14195 Berlin, Germany

[2]Scientific Computing Laboratory, Institute of Physics Belgrade, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

[3]Hanse-Wissenschaftskolleg, Lehmkuhlenbusch 4, 27753 Delmenhorst, Germany

[4]Department of Physics, Technische Universität Kaiserslautern, Erwin-Schrödinger-Straße, 67663 Kaiserslautern, Germany

Perturbation theory

- **Gross-Pitaevskii equation**

$$\left[-\frac{\hbar^2\Delta}{2m}-\mu+U(\mathbf{x})+\int d^3\mathbf{x}'\psi^*(\mathbf{x}')\psi(\mathbf{x}')V(\mathbf{x}-\mathbf{x}')\right]\psi(\mathbf{x})=0$$

  – Disorder potential $U(\mathbf{x})$, disorder ensemble average $\langle\bullet\rangle$ [1–7]

$$\langle U(\mathbf{x})\rangle=0 \quad\text{and}\quad \langle U(\mathbf{x})U(\mathbf{x}')\rangle=R(\mathbf{x}-\mathbf{x}')$$

  – Interaction potential $V(\mathbf{x})$

- **Perturbation expansion** [8]

$$\psi(\mathbf{x})=\psi_0+\psi_1(\mathbf{x})+\psi_2(\mathbf{x})+\dots$$

- **Condensate depletion**
  – Particle density $n=\langle\psi(\mathbf{x})^2\rangle=\psi_0^2+\langle\psi_1(\mathbf{x})^2\rangle+2\psi_0\langle\psi_2(\mathbf{x})\rangle+\dots$
  – Condensate density $n_0=\langle\psi(\mathbf{x})\rangle^2=\psi_0^2+2\psi_0\langle\psi_2(\mathbf{x})\rangle+\dots$
  – Depletion

$$n-n_0=n\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{R(\mathbf{k})}{\left[\frac{\hbar^2k^2}{2m}+2nV(\mathbf{k})\right]^2}+\dots$$

- **Equation of state**
  After renormalization [5]

$$\mu=nV(\mathbf{k}=0)+4n\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{V(\mathbf{k})R(\mathbf{k})\left(\frac{\hbar^2k^2}{2m}+nV(\mathbf{k})\right)}{\frac{\hbar^2k^2}{2m}\left[\frac{\hbar^2k^2}{2m}+2nV(\mathbf{k})\right]^2}+\dots$$

- **Superfluid Density**
  – Moving disorder, time-dependent Gross-Pitaevskii equation:

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+U(\mathbf{r}-\mathbf{v}t)+\int d^3\mathbf{r}'V(\mathbf{r}-\mathbf{r}')\Psi^*(\mathbf{r}',t)\Psi(\mathbf{r}',t)\right]\Psi(\mathbf{r},t)$$

  – Momentum density: $\langle\mathbf{j}\rangle=\frac{1}{2i}\langle\Psi^*\nabla\Psi-\Psi\nabla\Psi^*\rangle$

$$\langle\mathbf{j}\rangle=\hat{n}_S\mathbf{k}_S+\hat{n}_N\mathbf{k}_U\,,\ \mathbf{k}_U=\frac{m}{\hbar}\mathbf{v}$$

$$n_{Nij}=4n\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{R(\mathbf{k})k_ik_j}{\mathbf{k}^2[\frac{\hbar^2\mathbf{k}^2}{2m}+2nV(\mathbf{k})]^2}+\dots$$

$$n_{Sij}=n\delta_{ij}-4n\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{R(\mathbf{k})k_ik_j}{\mathbf{k}^2[\frac{\hbar^2\mathbf{k}^2}{2m}+2nV(\mathbf{k})]^2}+\dots$$

  – Isotropic case, $V(\mathbf{k})=V(|\mathbf{k}|)$ and $R(\mathbf{k})=R(|\mathbf{k}|)$:

$$n-n_s=\frac{4}{3}(n-n_0)$$

- **Sound velocity**
  – Two-fluid model: superfluid component (n_S, $\mathbf{v}_S$) and normal fluid component (n_N, $\mathbf{v}_N=\mathbf{0}$)
  – Linearization of hydrodynamic equations for superfluid leads to dispersion relation in long-wavelength limit:

$$c_{\mathbf{q}}^2=\frac{1}{m}\frac{\partial\mu}{\partial n}\,\mathbf{q}^T\hat{n}_S\mathbf{q}$$

Dipolar interaction and Lorentz-correlated disorder

BEC of atoms with magnetic dipole: ^{52}Cr [10] and ^{87}Rb [11], $C_{\rm dd}=\mu_0m^2$
BEC of molecules with electric dipole: ^{41}K^{87}Rb [12], $C_{\rm dd}=4\pi d^2$

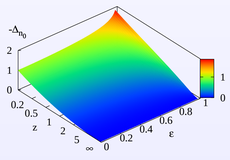
$$R(\mathbf{k})=\frac{R}{1+\sigma^2k^2},\ V(\mathbf{k})=g\left[1+\varepsilon_{\rm dd}(3\cos^2\theta_{\mathbf{k},\mathbf{m}}-1)\right],\ \varepsilon_{\rm dd}=\frac{C_{\rm dd}}{3g}$$

$\Delta_A=\frac{A/A(R=0)-A_d}{n_{\rm HM}/n}$, $n_{\rm HM}=\frac{R}{8\sqrt{2}\pi\xi^3ng^2}$ [1], $z=\frac{\sqrt{2}\sigma}{\xi}$, $\xi=\sqrt{\frac{\hbar^2}{2mng}}$, $T(x)=\frac{\arctan\sqrt{x}}{\sqrt{x}}$.

- **Condensate depletion**

$$-\Delta_{n_0}=\frac{(1-c)^2(1-\lambda)}{8c\delta}\left[\frac{1+c}{1+c\lambda}-(1-c)T(c\lambda)\right]$$

$$\delta=\sqrt{1-\varepsilon_{\rm dd}},\ \lambda=\frac{2}{3}\frac{1+\varepsilon_{\rm dd}-\delta\sqrt{1+2\varepsilon_{\rm dd}}}{\varepsilon_{\rm dd}},\ c=\frac{z\delta-1}{z\delta+1}$$

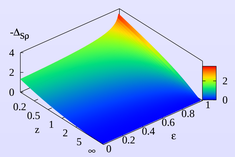


  – decreases as z increases
  – increases as $\varepsilon_{\rm dd}$ increases
  – diverges for $\varepsilon_{\rm dd}\to1$

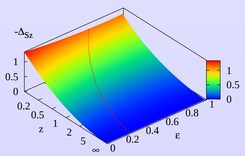
- **Equation of state**

  – μ is θ dependent
  – correction Δ_μ is θ independent
  – decreases as both z and $\varepsilon_{\rm dd}$ increase

- **Superfluid depletion**
  – Perpendicular component
    ∗ decreases as z increases
    ∗ increases as $\varepsilon_{\rm dd}$ increases
    ∗ diverges for $\varepsilon_{\rm dd}\to1$
    ∗ larger than condensate depletion



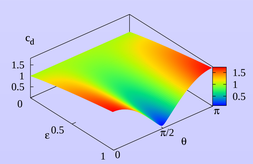
  – Parallel component
    ∗ decreases as both z and $\varepsilon_{\rm dd}$ increase
    ∗ smaller than condensate depletion for $\varepsilon_{\rm dd}>\varepsilon_{\rm dd,crit}(z)$ (red line)
    ∗ effective interaction strength [8]
    ∗ finite localization time [8,13]

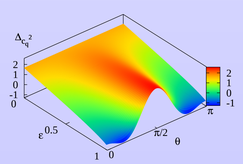


- **Sound velocity**

$c_{\mathbf{q}}^2=c_0^2\left(c_d(\mathbf{q})^2+\frac{n_{\rm HM}}{n}\Delta_{c_{\mathbf{q}}^2}\right)$,
$\theta=\angle(\mathbf{q},\mathbf{e}_z)$,

c_d - sound anisotropy for clean case





Anisotropic Lorentz-correlated disorder

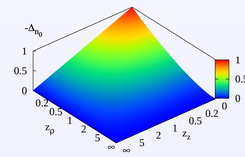
Laser speckles [14]: tunable disorder
Wire traps [15,16]: unwanted disorder

$$R(\mathbf{k})=\frac{R}{1+\sigma_\rho^2k_\rho^2+\sigma_z^2k_z^2},\ V(\mathbf{k})=g$$

- **Condensate depletion**

$$-\Delta_{n_0}=\frac{1}{(z_\rho-1)(z_\rho+1)}\left[\frac{2z_\rho^2}{(z_\rho+1)(z_\rho+z_z)}T\left(\frac{z_\rho-1}{z_\rho+1}\cdot\frac{z_z-z_\rho}{z_z+z_\rho}\right)-\frac{1}{z_z+1}\right]$$

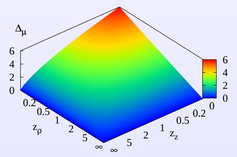
$$-\Delta_{n_0}\begin{cases}=1, & z_\rho=0 \text{ and } z_z=0\\ =0, & z_\rho\to\infty \text{ or } z_z\to\infty\\ \text{monotonously decreasing}\end{cases}$$



- **Equation of state**

$$\Delta_\mu=\begin{cases}6, & z_\rho=0 \text{ and } z_z=0\\ 0, & z_\rho\to\infty \text{ or } z_z\to\infty\end{cases}$$

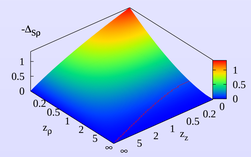
monotonously decreasing for all z_ρ and z_z

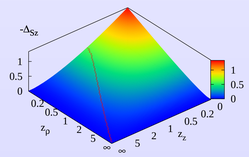


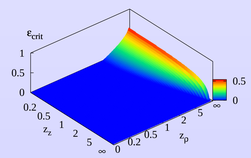
- **Superfluid depletion**

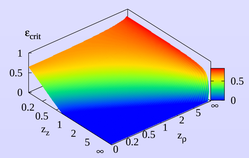
$$(n_S)_{i\neq j}=0,\quad -\Delta_{S_i}=\begin{cases}-\frac{4}{3}\Delta_d, & z_\rho=z_z\\ 0, & z_\rho\to\infty \text{ or } z_z\to\infty\end{cases}$$

monotonically decreasing, red line for $\Delta_{S_i}=\Delta_{n_0}$.



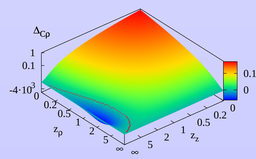


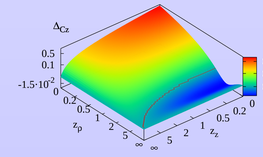




- **Sound velocity** $c_{\mathbf{q}}^2=c_\rho^2\sin^2\theta+c_z^2\cos^2\theta$

$$\Delta_{c_i}=\begin{cases}\frac{5-z}{6(1+z)^3}, & z=z_\rho=z_z\\ 0, & z_\rho\to\infty \text{ or } z_z\to\infty\end{cases}$$





Dipolar Bose-Einstein Condensates in Weak Anisotropic Disorder Potentials

Branko Nikolić[1,2], Antun Balaž[2], and Axel Pelster[3,4]

[1]Department of Physics, Freie Universität Berlin, Fabeckstraße 60, 14195 Berlin, Germany

[2]Scientific Computing Laboratory, Institute of Physics Belgrade, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia

[3]Hanse-Wissenschaftskolleg, Lehmkuhlenbusch 4, 27753 Delmenhorst, Germany

[4]Department of Physics, Technische Universität Kaiserslautern, Erwin-Schrödinger-Straße, 67663 Kaiserslautern, Germany

References

[1] K. Huang and H. F. Meng. Hard-sphere Bose gas in random external potentials. *Phys. Rev. Lett.* **69**, 644 (1992).

[2] S. Giorgini, L. Pitaevskii, and S. Stringari. Effects of disorder in a dilute Bose gas. *Phys. Rev. B* **49**, 12938 (1994).

[3] M. Kobayashi and M. Tsubota. Bose-Einstein condensation and superfluidity of a dilute Bose gas in a random potential. *Phys. Rev. B* **66**, 174516 (2002).

[4] A. V. Lopatin and V. M. Vinokur. Thermodynamics of the Superfluid Dilute Bose Gas with Disorder. *Phys. Rev. Lett.* **88**, 235503 (2002).

[5] G. M. Falco, A. Pelster, and R. Graham. Thermodynamics of a Bose-Einstein condensate with weak disorder. *Phys. Rev. A* **75**, 063619 (2007).

[6] R. Graham and A. Pelster, in *Path Integrals – New Trends and Perspectives: Proceedings of the 9th International Conference*, edited by W. Janke and A. Pelster (World Scientific, Singapore, 2008), p. 376.

[7] C. Gaul and C. A. Müller. Bogoliubov excitations of disordered Bose-Einstein condensates. *Phys. Rev. A* **83**, 063629 (2011)

[8] C. Krumnow and A. Pelster. Dipolar Bose-Einstein condensates with weak disorder. *Phys. Rev. A* **84**, 021608(R) (2011).

[9] P. Muruganandam and S. K. Adhikari. Anisotropic sound and shock waves in dipolar Bose-Einstein condensate. *Phys. Lett. A* **376**, 480 (2012).

[10] A. Griesmaier, J. Werner, S. Hensler, J. Stuhler, and T. Pfau. Bose-Einstein Condensation of Chromium. *Phys. Rev. Lett.* **94**, 160401 (2005).

[11] M. Vengalattore, S. R. Leslie, J. Guzman, and D. M. Stamper-Kurn. Spontaneously Modulated Spin Textures in a Dipolar Spinor Bose-Einstein Condensate *Phys. Rev. Lett.* **100**, 170403 (2008).

[12] K. Aikawa, D. Akamatsu, J. Kobayashi, M. Ueda, T. Kishimoto and S. Inouye. Toward the production of quantum degenerate bosonic polar molecules, ^{41}K^{87}Rb. *New J. Phys.* **11**, 055035 (2009).

[13] R. Graham and A. Pelster. Order via nonlinearity in randomly confined Bose gases. *Int. J. Bifur. Chaos* **19**, 2745 (2009).

[14] J. Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugan, D. Clément, L. Sanchez-Palencia, P. Bouyer and A. Aspect. Direct observation of Anderson localization of matter waves in a controlled disorder. *Nature* **453**, 891 (2008).

[15] P. Krüger, L. M. Andersson, S. Wildermuth, S. Hofferberth, E. Haller, S. Aigner, S. Groth, I. Bar-Joseph and J. Schmiedmayer. Potential roughness near lithographically fabricated atom chips. *Phys. Rev. A* **76**, 063621 (2007).

[16] J. Fortágh and C. Zimmermann. Magnetic microtraps for ultracold atoms. *Rev. Mod. Phys.* **79**, 235 (2007).

Financial support: This work was supported in part by DAAD - German Academic and Exchange Service under project NAD-BEC, Ministry of Education and Science of the Republic of Serbia under projects No. ON171017 and NAD-BEC, and by the European Commission under EU FP7 projects PRACE-2IP, PRACE-3IP, HP-SEE and EGI-InSPIRE.