Kibble-Zurek Mechanism (KBM)

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Topological defects 101

- Superfluid vortices (e.g. helium, superconductors, BKT)
- Domain walls
- Screw dislocations in crystals
- Magnetic monopoles
- Cosmic strings
- ….

Key features:

- Robust against small perturbations
- “Felt” from far away

Phase transition as symmetry breaking

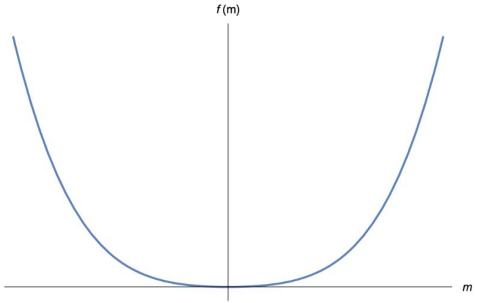


Figure 1: Free energy when $T > T_c$

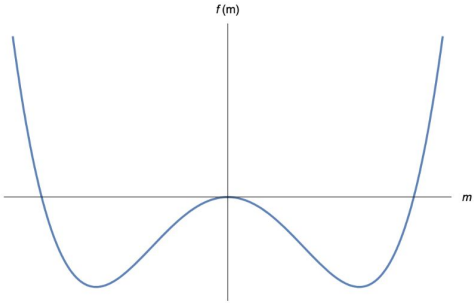
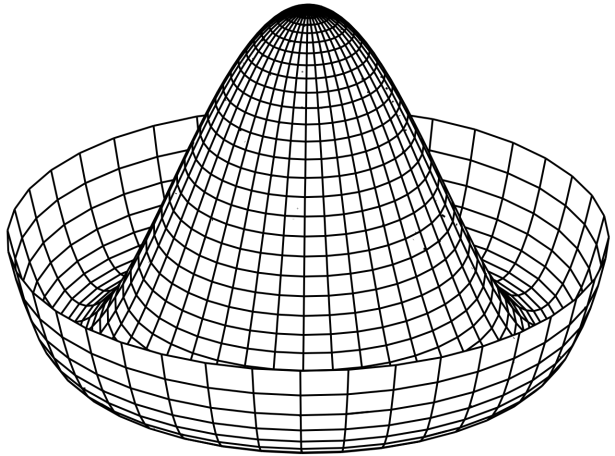


Figure 2: Free energy when $T < T_c$



Source: D. Tong, Cambridge Lecture Notes on Theoretical Physics

At different points of space the symmetry broken phase can end up in different equivalent minima. (in principle metastable but equilibration is suppressed by the system's volume)

Terminology

A defect is a feature of an order parameter field configuration.

- Base space – the space where our system lives (often 1D, 2D, or 3D Euclidean space)
- Target space – the space of possible values of the order parameter (e.g. 1-sphere for an angle variable, complex plane, space of unit vectors, symmetric tensors, etc.)

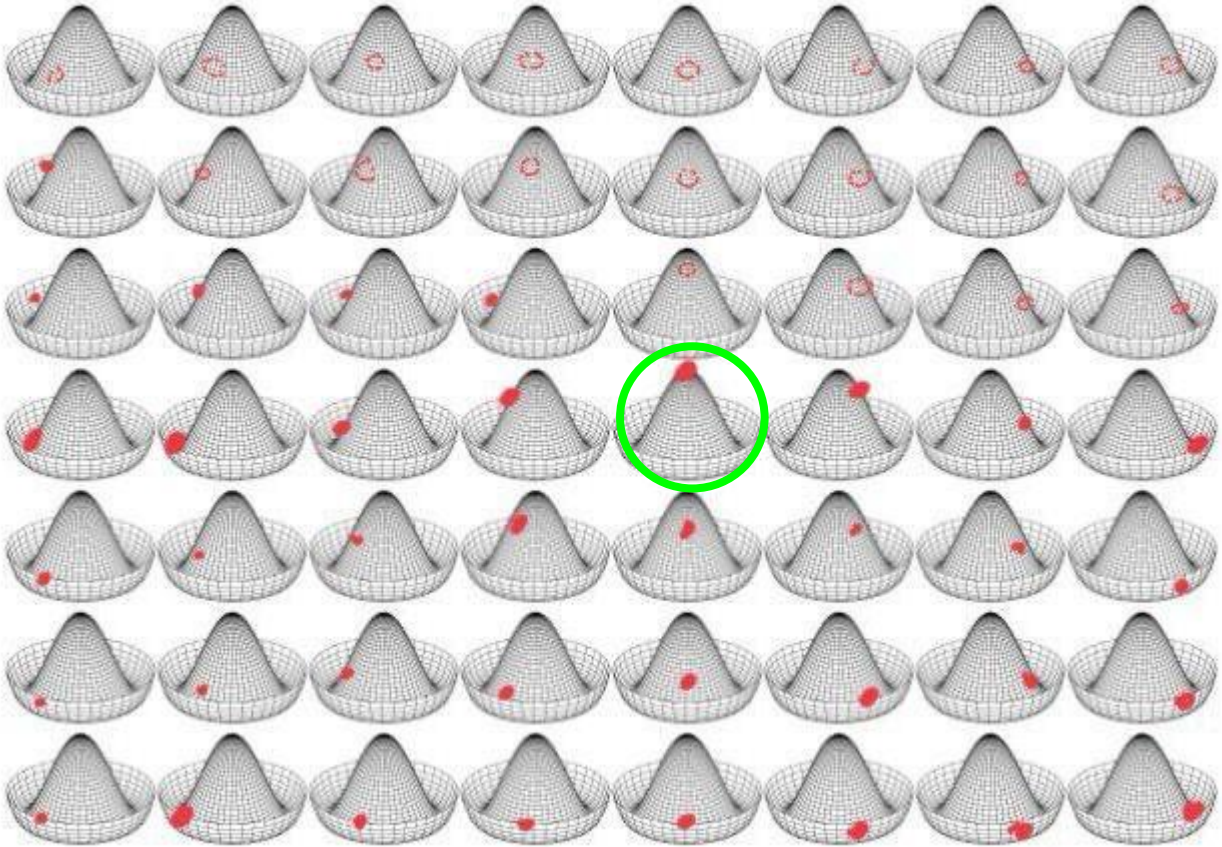
The field is a mapping from the base space to the target space. E.g. electrostatic field is a mapping from 3D to 3D.

Properties of the two spaces determine what kinds of defects can exist (homotopy theory). Homotopy group describes the topological properties of the order parameter space. The nature of the defect is determined by the homotopy group π_p where $p+d+1=D$.

Defects can in principle have any dimension smaller than dimension of the base space.

Example 1: quantum vortex

- The order parameter varies smoothly everywhere
- When $|\Psi|=0$, phase undetermined
- There is an energy cost (gradient terms not shown)
- The unfavourable Ψ near the core is locked by topology



Example 1: quantum vortex

- As we go around a vortex and track the phase of the order parameter, we find that it changes by 2π
- For *any* field configuration the change of phase has to be $2\pi n$.
- Small variations in the field values will not change *n* since it is discrete
- The value of *n* can be inferred from loops that are far from the vortex core

Summary:

- Causally disconnected regions of space can transition into different equivalent minima
- This might enforce formation of an (energetically unfavourable) defect
- Defects are very stable

Example 2: magnetic monopoles

- Order parameter – magnetic vector potential
- Invariant quantity – magnetic flux through a contour

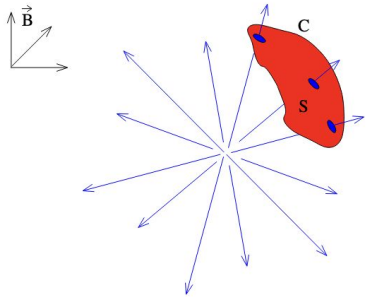


Figure 85: Integrating over S...

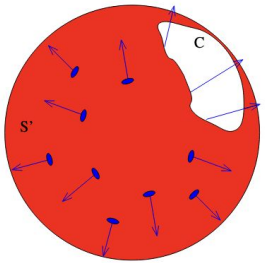


Figure 86: ...or over S'.

Source: D. Tong, Cambridge Lecture Notes on Theoretical Physics

Example 2: magnetic monopoles

Production in the early universe

Cosmic inflation



Zel’dovich 1978

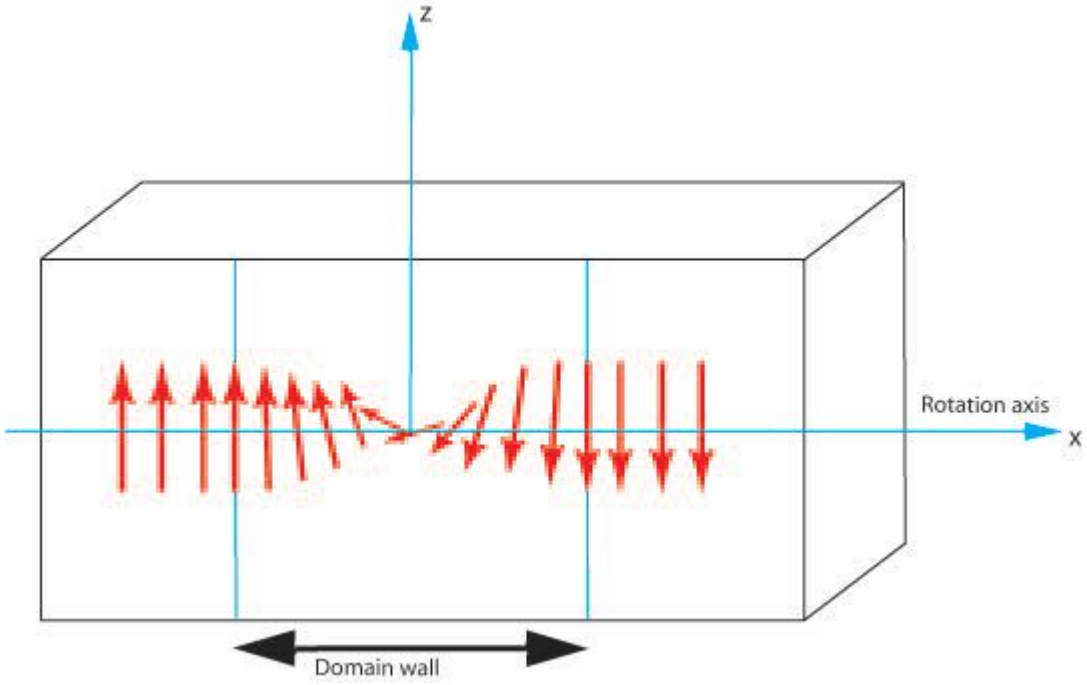


Preskill 1979



Guth 1979

Example 3: domain walls

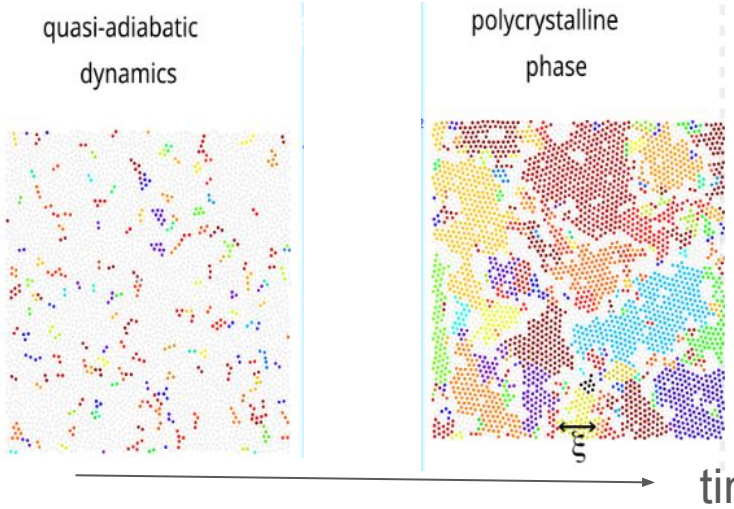


Kibble-Zurek: motivation

- When a system undergoes a continuous phase transition at a finite rate, topological defects can form
- This is true for both quantum and thermal phase transitions

- We observe that when a system undergoes a phase transition due to a quenching, the resulting state is dependent on the speed of this quenching

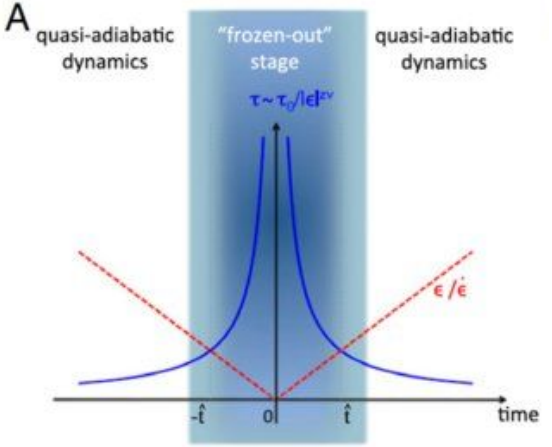
- We will see a different number of defects
- Faster quench = more topological defects



Kibble-Zurek: heuristic example

- Say we have a system which undergoes a continuous phase transition
- In particular, say we have a paramagnetic state which, under a global quench, can result in a ferromagnetic state
- If we quench:
    - Infinitely slowly: the system should end up in one of the ordered ferromagnetic ground states, with no defects
    - Infinitely quickly: our final system will still be disordered
  - So for any intermediate quenching time we expect there to be an intermediate state with order and disorder, ie with a finite number of defects

Kibble Zurek mechanism: conceptual idea



- At the critical point, our control parameter is zero
- During the transition we can divide the system into
- different regimes, which will be dependent on
- the value of the control parameter at that time
- In particular, if we have a linear quench then:
  - 1st equilibrium: adiabatic system is in a particular state and is undergoing the transition.
  - Freeze-out: time that system takes to respond to the changes in control parameter is greater than the time in which these changes occur, so non-equilibrium state is formed
  - 2nd equilibrium: post freeze-out the system forms a new equilibrium state that is different to the first

- Correlation length and equilibration time increase as the system approaches critical point

Kibble-Zurek: mathematical formalism

- Control parameter: ε(t)
- Correlation length $\xi = \xi_0 / \varepsilon$
- Equilibrium time $\tau = \tau_0 / \varepsilon$
- Linear quench: $\varepsilon = t / \tau_Q$
- Freeze-out time is $\hat{t}$
- At the freeze-out time the rate at which we quench is comparable to the response time of the system $\tau(\hat{t}) = \hat{t}$
-

$$\hat{\xi} := \xi(\hat{t}) = \xi_0 \sqrt{\frac{\tau_Q}{\tau_0}} \longrightarrow \nu_{KZM} \sim \frac{\alpha}{\hat{\xi}} \Rightarrow \boxed{\nu_{KZM} \propto \frac{1}{\sqrt{\tau_Q}}}$$

Kibble-Zurek: how do defects form?

- Recall that we have two different equilibrium states, before and after freeze-out
- These will have different order parameters
- At freeze-out, system stops evolving in response to changes in $\varepsilon(t)$
- Since correlation length is frozen in, the system can only form ordered states on the scale of ξ
- After freeze-out different regions settle into different order parameter configurations
- Defects will form at the boundaries of these regions

SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Quantum phase transition dynamics in the two-dimensional transverse-field Ising model

Markus Schmitt[1]*, Marek M. Rams[2]*, Jacek Dziarmaga[2], Markus Heyl[3,4], Wojciech H. Zurek[5]

The quantum Kibble-Zurek mechanism (QKZM) predicts universal dynamical behavior near the quantum phase transitions (QPTs). It is now well understood for the one-dimensional quantum matter. Higher-dimensional systems, however, remain a challenge, complicated by the fundamentally different character of the associated QPTs and their underlying conformal field theories. In this work, we take the first steps toward theoretical exploration of the QKZM in two dimensions for interacting quantum matter. We study the dynamical crossing of the QPT in the paradigmatic Ising model by a joint effort of modern state-of-the-art numerical methods, including artificial neural networks and tensor networks. As a central result, we quantify universal QKZM behavior close to the QPT. We also note that, upon traversing further into the ferromagnetic regime, deviations from the QKZM prediction appear. We explain the observed behavior by proposing an extended QKZM taking into account spectral information as well as phase ordering. Our work provides a testing platform for higher-dimensional quantum simulators.

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Why observe the 2D Ising model ?

- 1D Ising model: no phase transition at finite temperature (DPT at T=0)
- 2D Ising model: continuous phase transition at finite temperature
- Interactions in 2D add complexity to topological defects
- Quantum phase transition at $g_c/J_c = 3.04438$

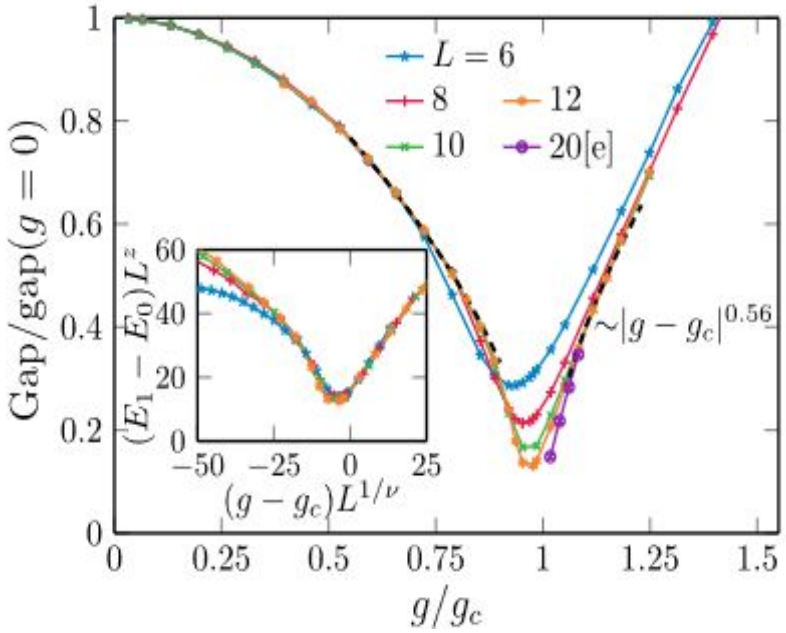
$$H(t) = -J(t) \sum_{\langle m,n \rangle} \sigma_m^z \sigma_n^z - g(t) \sum_{m=1}^{L^2} \sigma_m^x$$

Quantum phase transitions

- QPT from disordered to ordered phase
- Dynamical crossing: quenching through QP leads to universal topological defects
- Control parameter $\varepsilon = g/g_c - 1$, distance from QPT
- At CP: correlation length diverges : $\xi \propto |\varepsilon|^{-\nu}$

  energy gap closes $\Delta E \propto \xi^{-z}$

- Fig.: Energy gap between ground state and first excited state (PBC)



Dynamics of the system

- System initially in a paramagnetic ground state (J/g=0)
- Two protocols: slow ramp across QCP and linearized quench: $\varepsilon(t)=t/\tau_Q$,
- t=0 corresponds to QCP in adiabatic limit, τ_Q sets speed of quench
- Correlation length ξ increases adiabatically as long as the ramp is slow
- Excitations of order parameter have limited speed of propagation v
- Sonic horizon: limit where ξ can't grow adiabatically

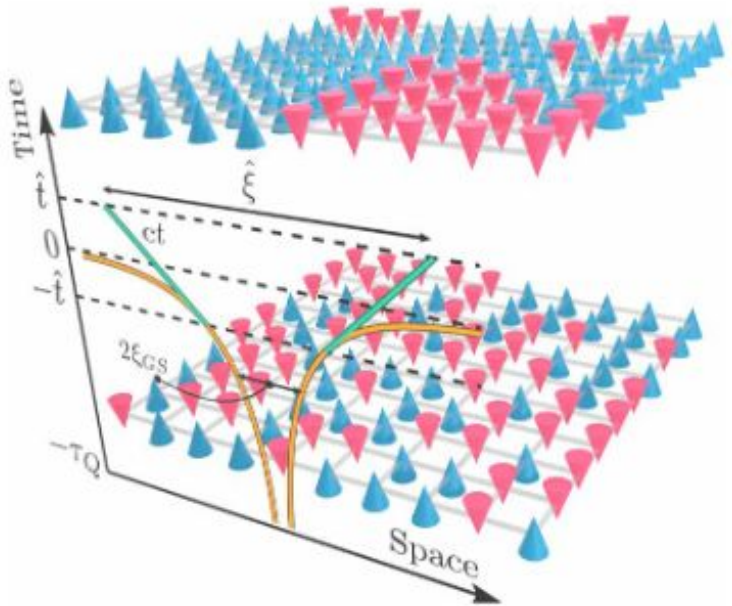
→topological defects / domain formation

characteristic time scale where dξ/dt exceeds v

$$\hat{t} \propto \tau_Q^{z\nu/(1+z\nu)}$$

Dynamics across a phase transition in 2D spin-1/2 model

- Initially in paramagnetic state (bottom)
- Slow ramp across QCP → quantum superposition of ferromagnetic domains (top)



Quantum Ising model in infinite lattice in TD limit

- 2 main markers for topological defects
- Ferromagnetic correlation function

$$C^{zz}(t,R) = \langle \psi(t) | \sigma_m^z \sigma_n^z | \psi(t) \rangle$$

- Excitation energy density (number of topological defects)

$$Q = \frac{1}{L^2}(\langle H(t) \rangle - E_0(t))$$

- Result: data collaps aligns with QKZM prediction
- Smooths ramp: provides clearer evidence of universal scaling behavior
- Linear quench: more topological defects, deviations from perfect scaling behaviour

