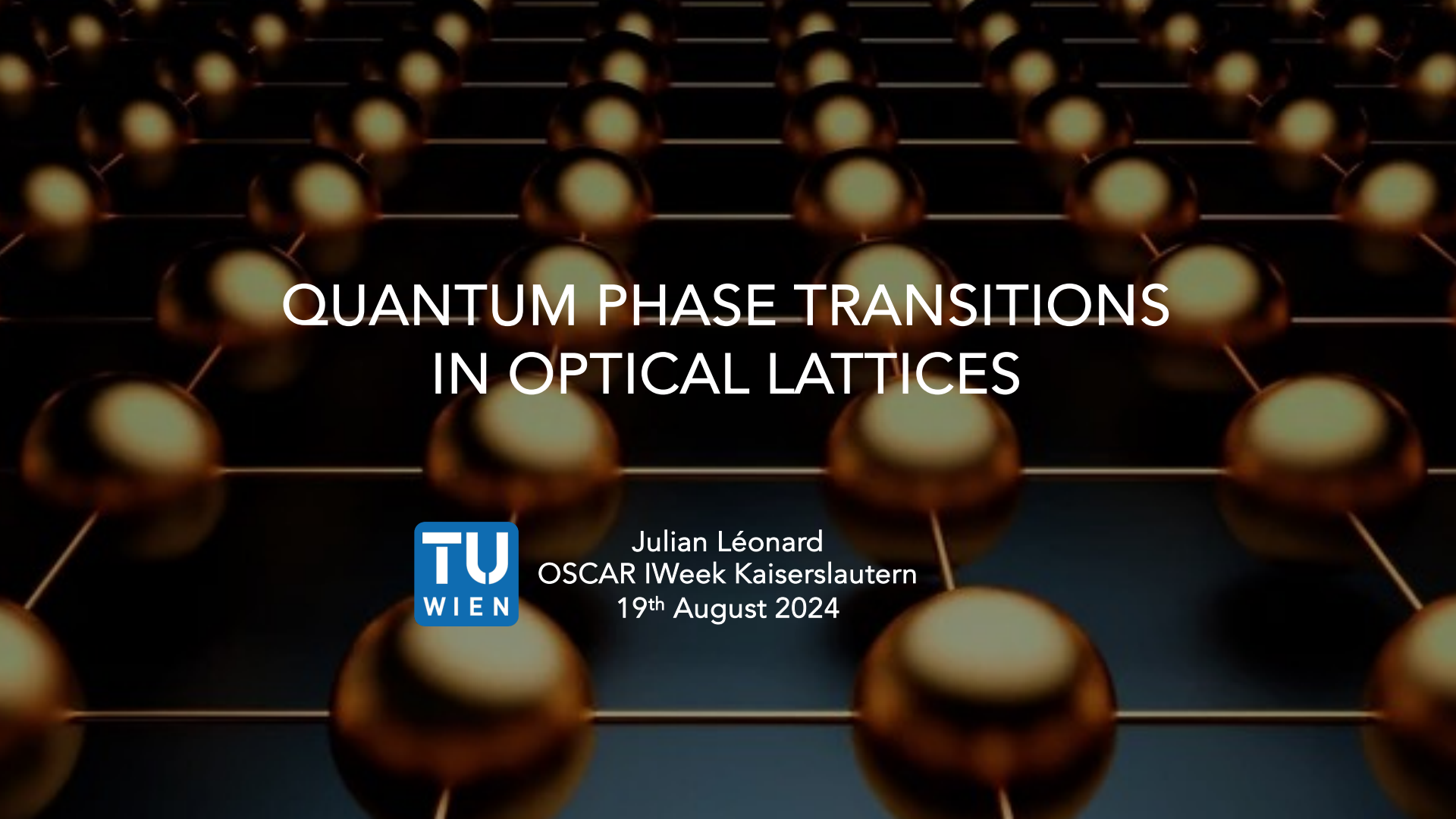


QUANTUM PHASE TRANSITIONS IN OPTICAL LATTICES

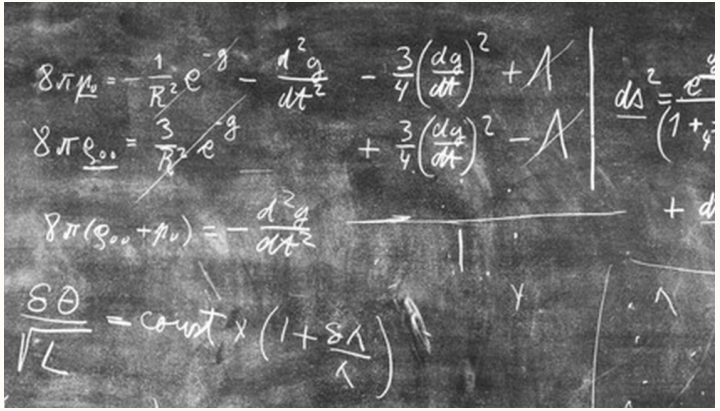
Julian Léonard

OSCAR IWeek Kaiserslautern

19th August 2024





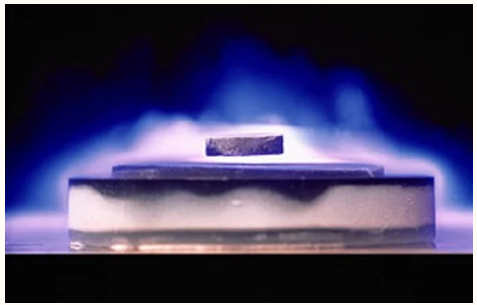






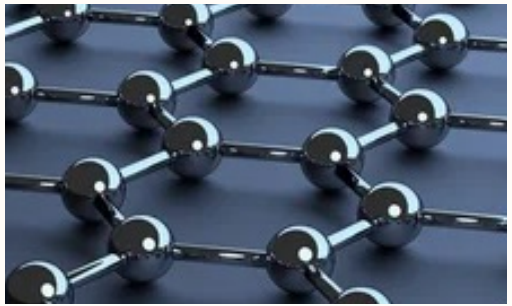
THE CHALLENGE OF MANY-BODY QUANTUM SYSTEMS

Understand and design many-body systems
One of the biggest challenges
of 21st century quantum physics



Technological relevance

- High-Tc superconductivity
- Magnetism
- Novel quantum sensors
- Quantum technologies



Fundamental interest

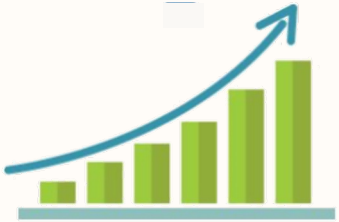
- Parameter changes
- Benchmark theories
- Many “simple“ models not solvable
- Discern different effects

WHICH PLATFORM?



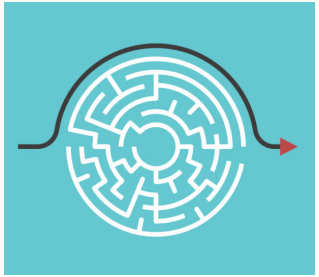
Building blocks

Identical, well understood



Scalable

Ability to connect many building blocks



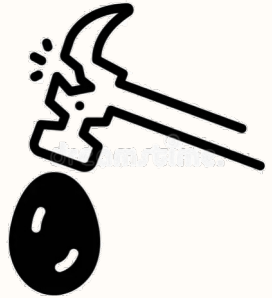
Simple

No behaviour beyond the computation



Controllable

Many control parameters, precise, independent



Robust

Reproducible results, insensitive to noise

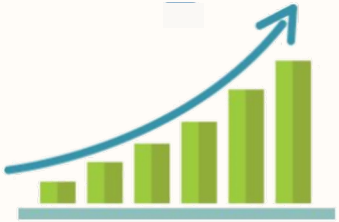
ATOMS ARE IDEAL PLATFORM



Building blocks

Identical, well understood

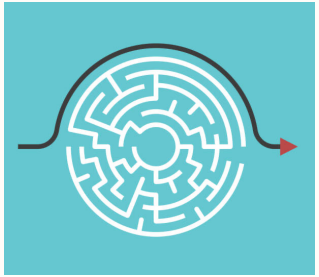
➔ exactly identical, atomic physics highly precise (~10^{-20})



Scalable

Ability to connect many building blocks

➔ possible to trap thousands of atoms



Simple

No behaviour beyond the computation

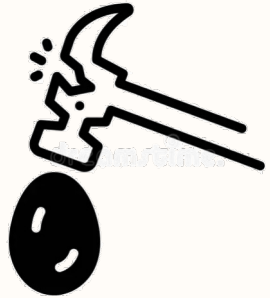
➔ isolated in vacuum



Controllable

Many control parameters, precise, independent

➔ optical control (motion, interactions,...)

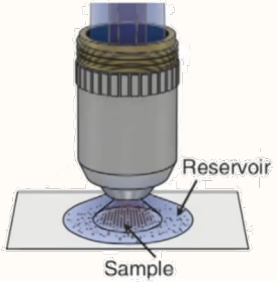


Robust

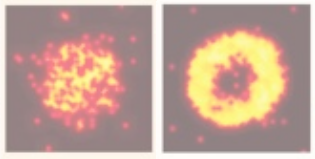
Reproducible results, insensitive to noise

➔ even at microscopic level

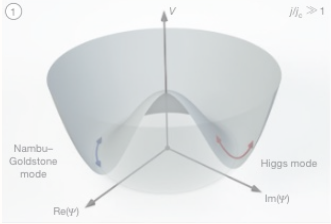
OUTLINE



Optical lattices



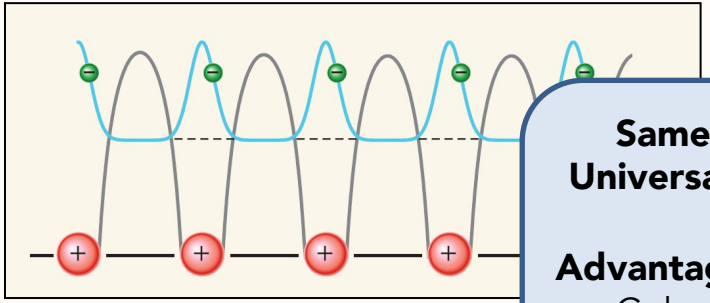
Superfluid-Mott insulator transition



Elementary excitations

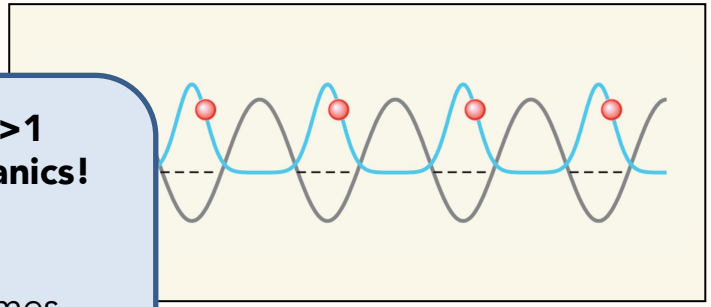
REALIZING ELECTRONIC SYSTEMS

Electrons in a crystal



Atoms in an optical potential

optical lattice



Same quantum regime: $\lambda/d > 1$
Universality of quantum mechanics!

Advantages

- Coherent: >1000 tunneling times
- Scalable: >1000 sites
- Fermionic and bosonic statistics
- Tunable interaction, temperature, doping, …
- Microscopic control

Lattice constant: ~Å

Densities: ~10^{25}/cm^3

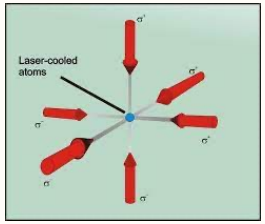
Temperatures: ~K

Lattice constant: ~μm

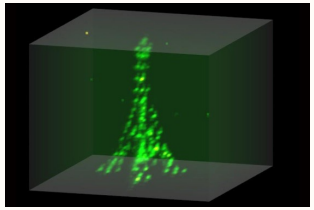
Densities: ~10^{14}/cm^3

Temperatures: ~pK

QUANTUM SIMULATION TOOLBOX

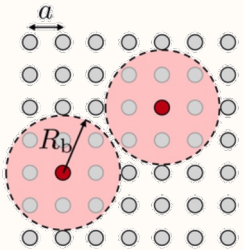


State preparation
Laser cooling
Optical pumping
…

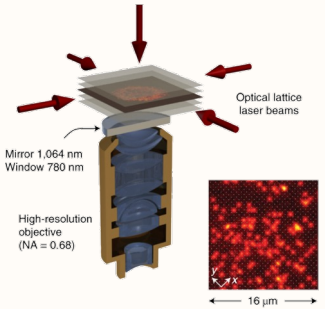


$$U_{dip} \propto -\alpha(\omega) I(\vec{r})$$

Potential control
Holographic beam shaping
Spin-dependent beams
Gauge fields
…



Interaction control
Feshbach resonance
Photon-mediated interactions
Dipoles, Rydberg,
…



State readout
Microscopy
Fluorescence imaging
Dispersive readout…

Key advantages
- Excellent coherence properties
- Easy to create large numbers of neutral atoms
- Easy to manipulate with light

Key challenges
- Bare atoms interact weakly
- Hard to obtain individual control over large numbers of atoms

BLOCH BANDS

A 3D optical lattice has the following potential:

$$V(x,y,z) = -V_x e^{-2\frac{y^2+z^2}{w_x^2}} \sin^2 kx - V_y e^{-2\frac{x^2+z^2}{w_y^2}} \sin^2 ky - V_z e^{-2\frac{x^2+y^2}{w_z^2}} \sin^2 kz$$
$$\approx -V_x \sin^2 kx - V_y \sin^2 ky - V_z \sin^2 kz + \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

From this we now want to derive a tight-binding Hamiltonian of the form:

$$\mathcal{H} = -t\,|i+1\rangle\langle i| - t\,|i\rangle\langle i+1| = \begin{pmatrix} 0 & -t \\ -t & 0 \end{pmatrix}$$

As an ansatz we use Bloch wave functions

$$\phi_{n,p}(x) = e^{ipx/\hbar} \cdot u_{n,p}(x)$$

which are composed of plane waves with quasi-momentum $p \in [-\pi/a, \pi/a]$ and the a function

$u_{n,p}(x) = u_{n,p}(x+a)$ With the periodicity of the lattice.

BLOCH BANDS

We introduce the Wannier functions: $$w_{n,i}(x) = w_n(x - x_i) = \int_{-\pi/a}^{\pi/a} \frac{dp}{2\pi} \phi_{n,p}(x) e^{-ix_i p}$$

Which are localized on the lattice sites x_i. By expanding the field operators $\psi(x) = \sum_{n,i} w^*_{n,i}(x) a_{n,i}$ in terms of the Wannier functions we obtain for the Hamiltonian:

$$\begin{aligned}
\mathcal{H} &= \int dx \psi^\dagger(x) \left[-\frac{\hbar^2}{2m}\partial_x^2 + V(x) \right] \psi(x) \\
&= \sum_{ijnm} a^\dagger_{n,i} a_{m,j} \int dx w_n(x - x_i) \left[-\frac{\hbar^2}{2m}\partial_x^2 + V(x) \right] w^*_m(x - x_j) \\
&= \sum_{ijn} a^\dagger_{n,i} a_{n,j} \int dx w_n(x - x_i) \left[-\frac{\hbar^2}{2m}\partial_x^2 + V(x) \right] w^*_n(x - x_j) \qquad \text{(only couplings within the same band n)} \\
&\approx \sum_{in} \left(a^\dagger_{n,i} a_{n,i+1} + a^\dagger_{n,i+1} a_{n,i} \right) \int dx w_n(x - x_i) \left[-\frac{\hbar^2}{2m}\partial_x^2 + V(x) \right] w^*_n(x - x_{i+1}) \qquad \text{(only nearest neighbour coupling)} \\
&\approx -t \sum_{\langle i,j \rangle} a^\dagger_i a_j . \qquad \text{(only lowest band n=1)}
\end{aligned}$$

BLOCH BANDS

To diagonalize the Hamiltonian we expand the annihilation operators as plane waves

$$a_i = \sum_k a_k e^{ikx_i}/\sqrt{N},$$

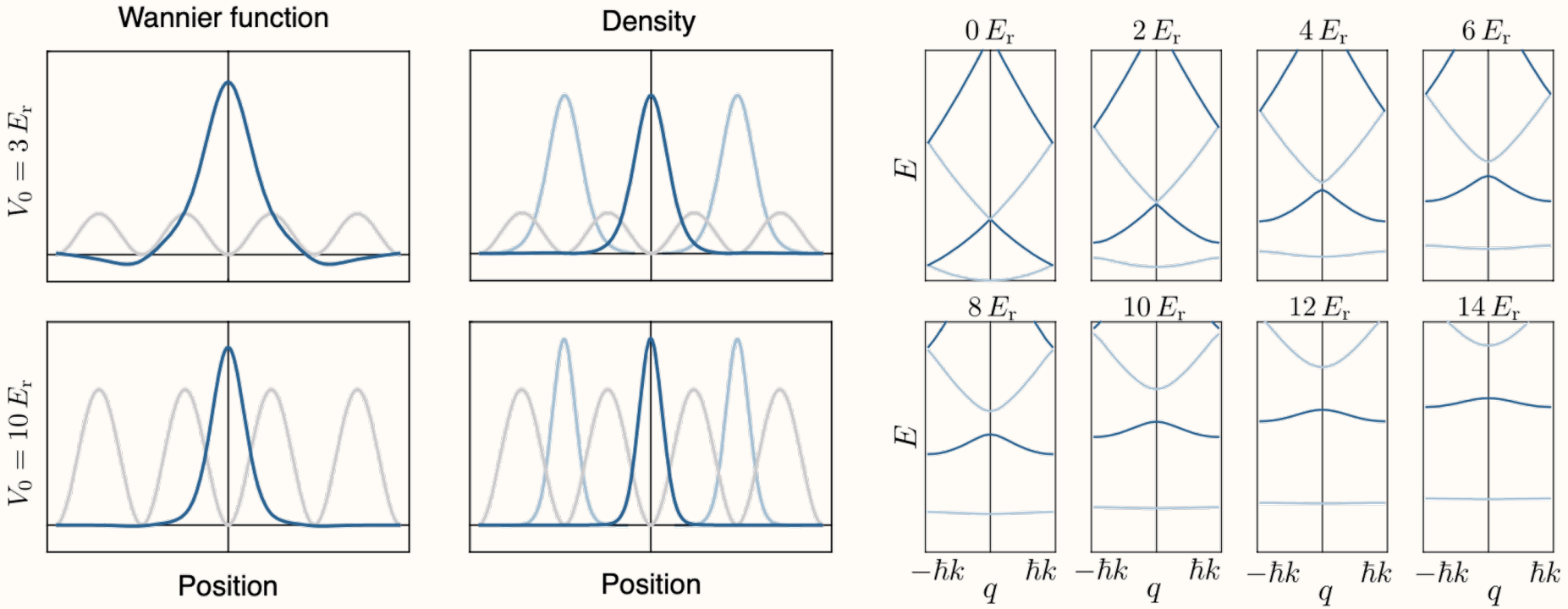
and obtain

$$\mathcal{H} = -t\sum_{\langle i,j\rangle} a_i^\dagger a_j = -\frac{t}{N}\sum_{k,k'} a_k^\dagger a_{k'} \sum_{\langle i,j\rangle} e^{ikx_i - ik'x_j} = -2t\sum_k \cos(ka) a_k^\dagger a_k = \sum_k \epsilon(k) a_k^\dagger a_k.$$

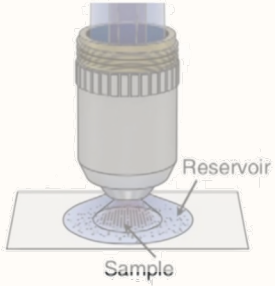
This means the lowest band has the dispersion relation:

$$\epsilon(k) = -2t\cos ka.$$

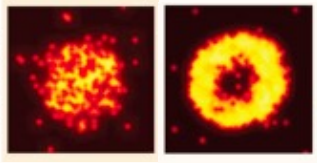
BLOCH BANDS


Wannier function
Density
$V_0 = 3\,E_\mathrm{r}$
$V_0 = 10\,E_\mathrm{r}$
Position
Position
$0\,E_\mathrm{r}$
$2\,E_\mathrm{r}$
$4\,E_\mathrm{r}$
$6\,E_\mathrm{r}$
$8\,E_\mathrm{r}$
$10\,E_\mathrm{r}$
$12\,E_\mathrm{r}$
$14\,E_\mathrm{r}$
E
E
$-\hbar k$
$\hbar k$
q

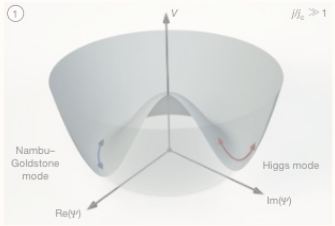
OUTLINE



Optical lattices



Superfluid-Mott insulator transition



Elementary excitations

BOSE-HUBBARD MODEL

VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

Cold Bosonic Atoms in Optical Lattices

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(Received 26 May 1998)

The dynamics of an ultracold dilute gas of bosonic atoms in an optical lattice can be described by a Bose-Hubbard model where the system parameters are controlled by laser light. We study the continuous (zero temperature) quantum phase transition from the superfluid to the Mott insulator phase induced by varying the depth of the optical potential, where the Mott insulator phase corresponds to a commensurate filling of the lattice ("optical crystal"). Examples for formation of Mott structures in optical lattices with a superimposed harmonic trap and in optical superlattices are presented. [S0031-9007(98)07267-6]

PACS numbers: 32.80.Pj, 03.75.Fi, 71.35.Lk

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i a_i^\dagger a_i \left(a_i^\dagger a_i - 1 \right) - \mu \sum_i a_i^\dagger a_i,$$

BOSE-HUBBARD MODEL

Now we include contact interactions:

$$\begin{aligned}\mathcal{H}_{\text{int}} &= \frac{4\pi\hbar^2 a_{\text{s}}}{2m}\int d\vec{r}\psi^\dagger(\vec{r})\psi^\dagger(\vec{r})\psi(\vec{r})\psi(\vec{r}) \\ &= \frac{4\pi\hbar^2 a_{\text{s}}}{2m}\sum_{i,j,l,m} a_i^\dagger a_j^\dagger a_k a_l \int d\vec{r} w_i(\vec{r})w_j(\vec{r})w_k^*(\vec{r})w_l^*(\vec{r}) \\ &= \sum_i a_i^\dagger a_i^\dagger a_i a_i \frac{4\pi\hbar^2 a_{\text{s}}}{2m}\int d\vec{r}|w_i(\vec{r})|^4 = \frac{U}{2}\sum_i a_i^\dagger a_i^\dagger a_i a_i.\end{aligned}$$

Together with hopping term, and including a chemical potential:

$$\mathcal{H} = -t\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i a_i^\dagger a_i\left(a_i^\dagger a_i - 1\right) - \mu\sum_i a_i^\dagger a_i,$$

Bose-Hubbard Hamiltonian
QPT: Two non-commuting terms

BOSE-HUBBARD DOUBLE WELL

Bose-Hubbard model for two particles on a double well:

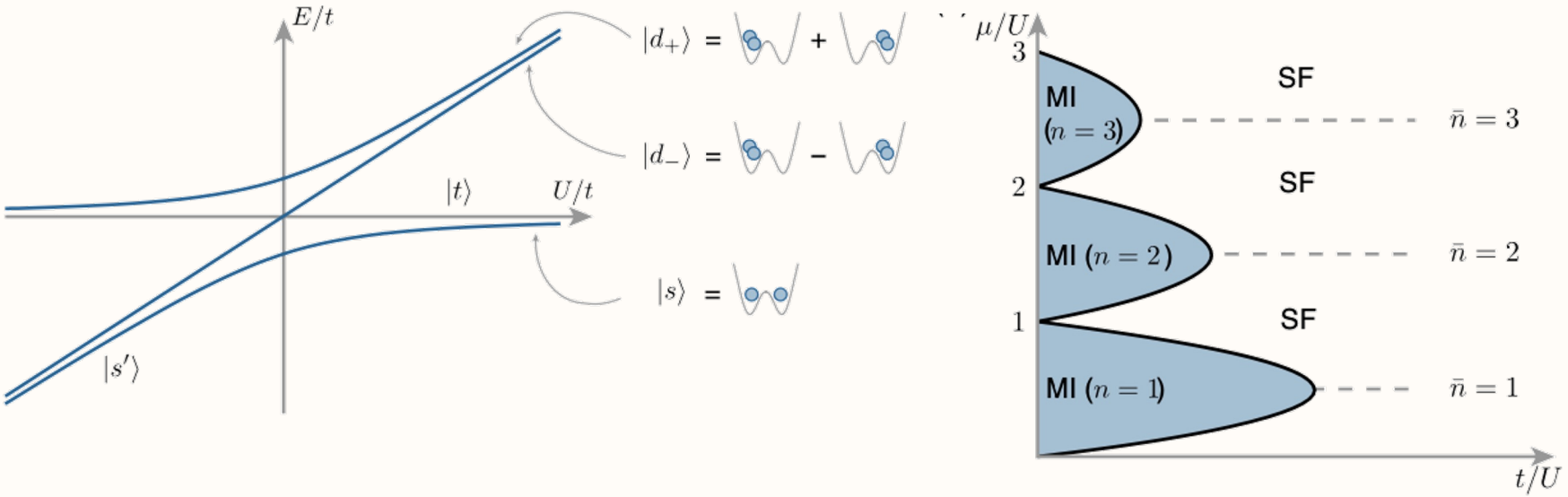
$$\mathcal{H} = -t\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) + \frac{U}{2}\sum_{i=\{1,2\}} a_i^\dagger a_i\left(a_i^\dagger a_i - 1\right) = \begin{pmatrix} 0 & -\sqrt{2}t & -\sqrt{2}t \\ -\sqrt{2}t & U & 0 \\ -\sqrt{2}t & 0 & U \end{pmatrix}$$

The eigenstates are

$$\begin{aligned} |1\rangle &= |d_-\rangle \\ |2\rangle &= |d_+\rangle + \frac{U + \sqrt{16t^2 + U^2}}{\sqrt{8}t}|s\rangle \\ |3\rangle &= |d_+\rangle + \frac{U - \sqrt{16t^2 + U^2}}{\sqrt{8}t}|s\rangle \end{aligned}$$

They are based on the singlons state $|s\rangle = |1,1\rangle$ and the symmetric and antisymmetric doublon states $|d_\pm\rangle = (|0,2\rangle \pm |2,0\rangle)/\sqrt{2}$.

BOSE-HUBBARD DOUBLE WELL



BOSE-HUBBARD MODEL

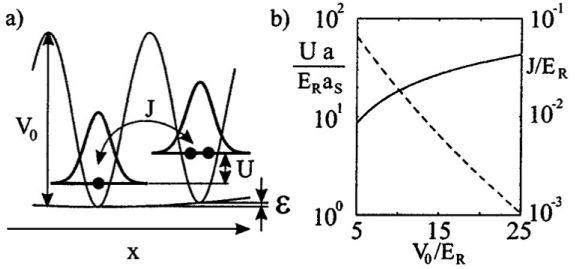


FIG. 1. (a) Realization of the BHM in an optical lattice (see text). The offset of the bottoms of the wells indicates a trapping potential V_T. (b) Plot of the scaled on site interaction U/E_R multiplied by a/a_s ($\gg 1$) (solid line; axis on left-hand side of graph) and J/E_R (dashed line; axis on right-hand side of graph) as a function of $V_0/E_R \equiv V_{x,y,z0}/E_R$ (3D lattice).

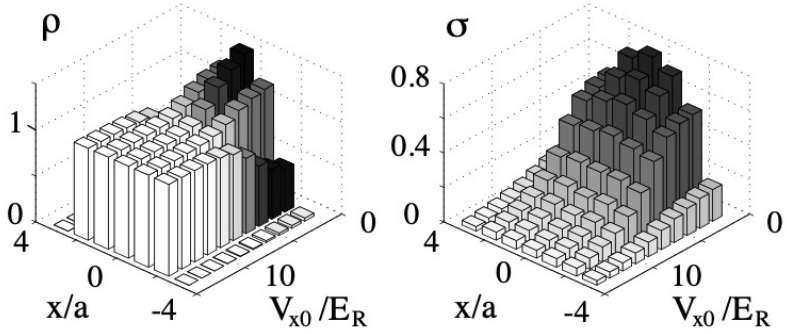


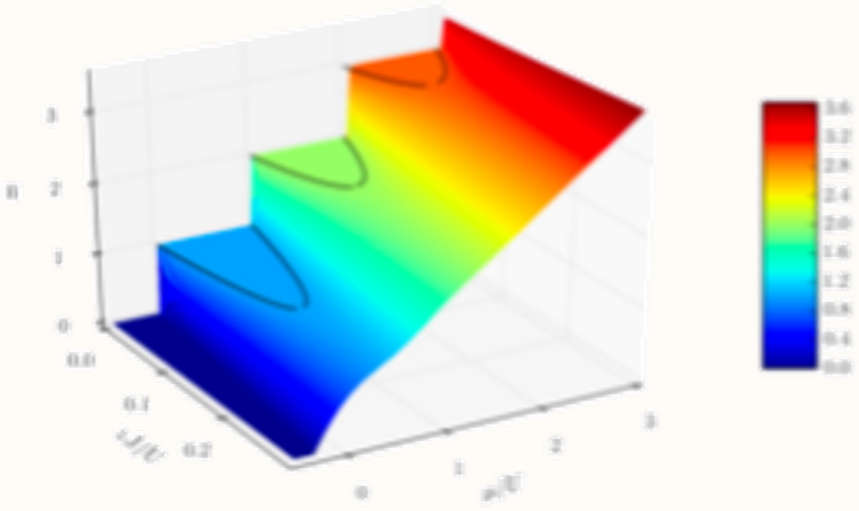
FIG. 3. Density ρ and fluctuations σ for the exact ground state in 1D for $N = 5$ atoms in a harmonic well as a function of V_{x0}/E_R for seven lattice cells. The parameters are $a_s/a = 1.1\ 10^{-2}$ (corresponding to Na and $\lambda = 514$ nm, $V_{y0} = V_{z0} = 40E_R$) and $V_T(x) = 0.06\ E_R(x/a)^2$.

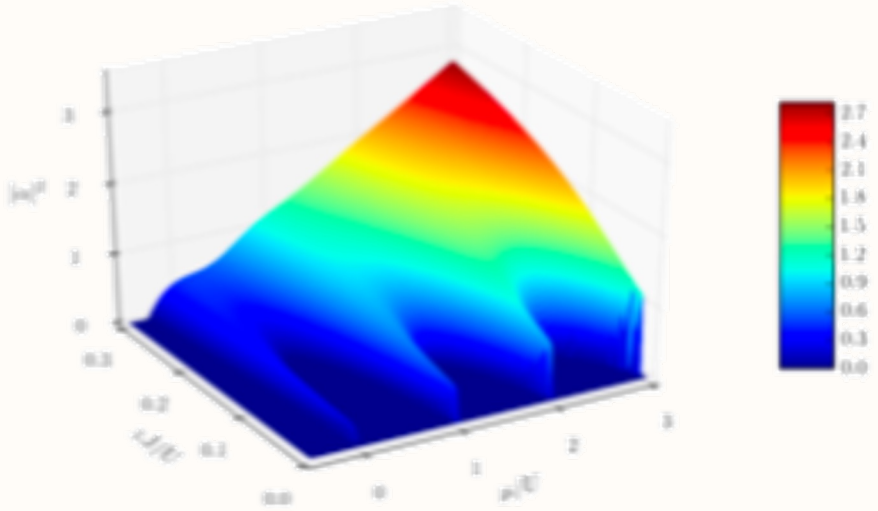
Both ground states are simple:

$$|\Psi_{\mathrm{SF}}\rangle_{U=0} \propto \left(\sum_{i=1}^{M} \hat{a}_i^\dagger\right)^N |0\rangle$$

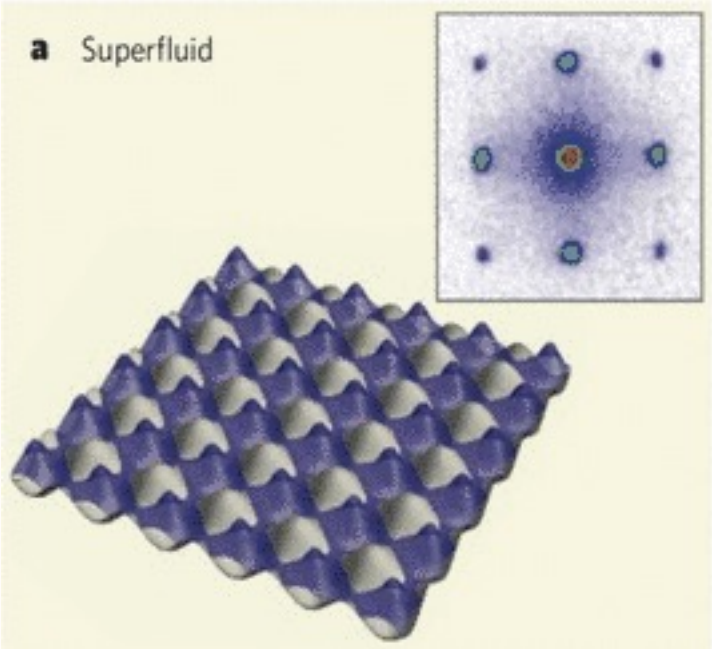
$$|\Psi_{\mathrm{MI}}\rangle_{J=0} \propto \prod_{i=1}^{M} (\hat{a}_i^\dagger)^n |0\rangle$$

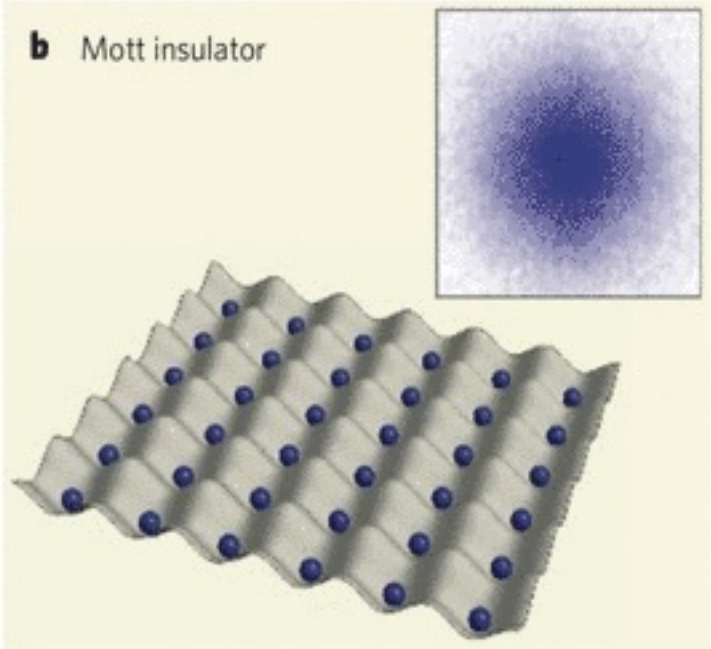
BOSONIC WEDDING CAKE





SUPERFLUID-MOTT INSULATOR TRANSITION





M. Greiner et al., *Nature* **415**, 39 (2002)

SUPERFLUID-MOTT INSULATOR TRANSITION

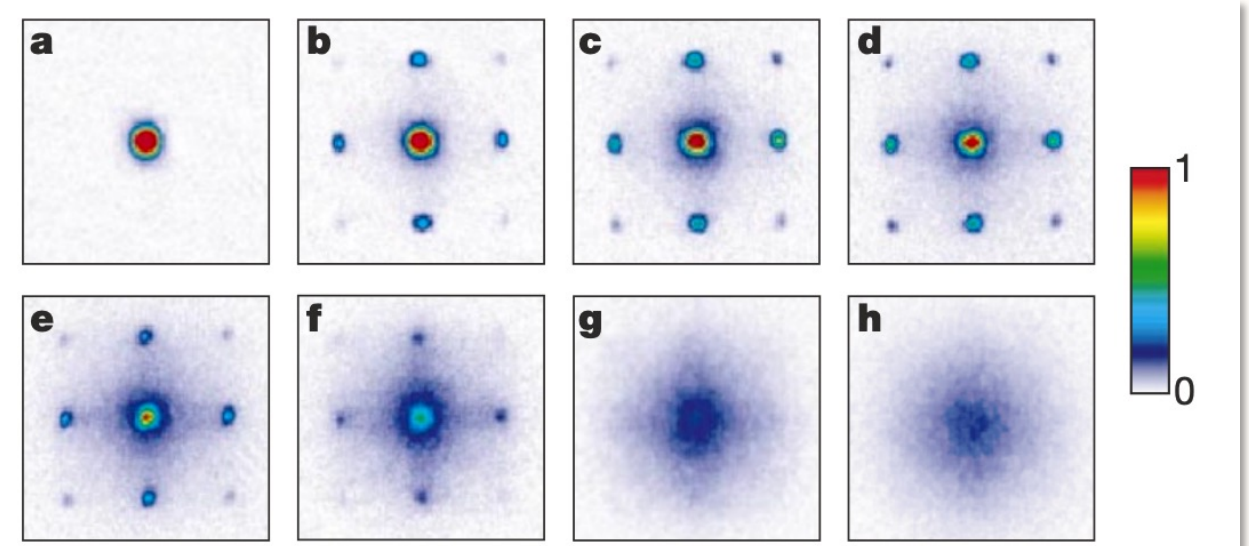
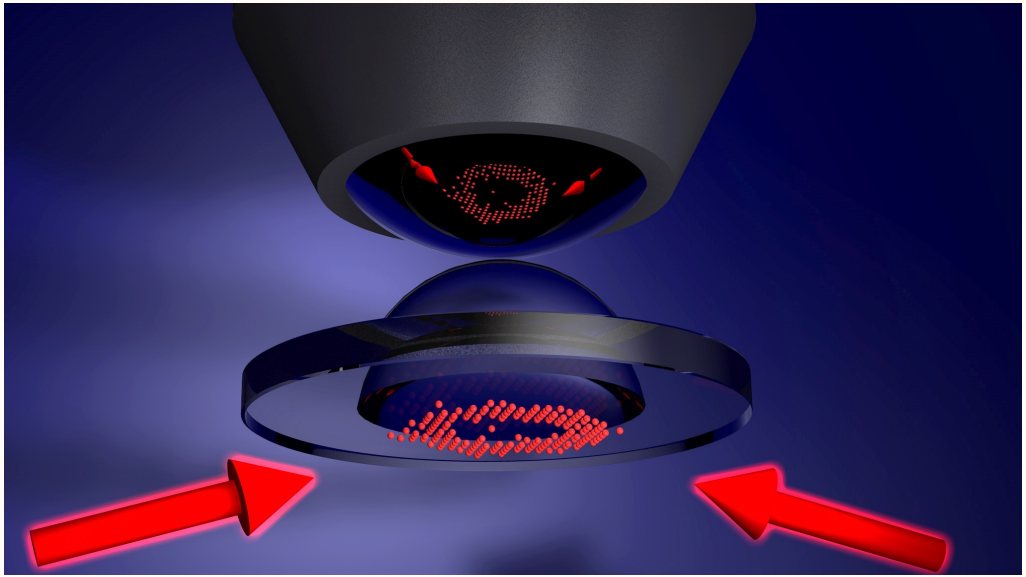
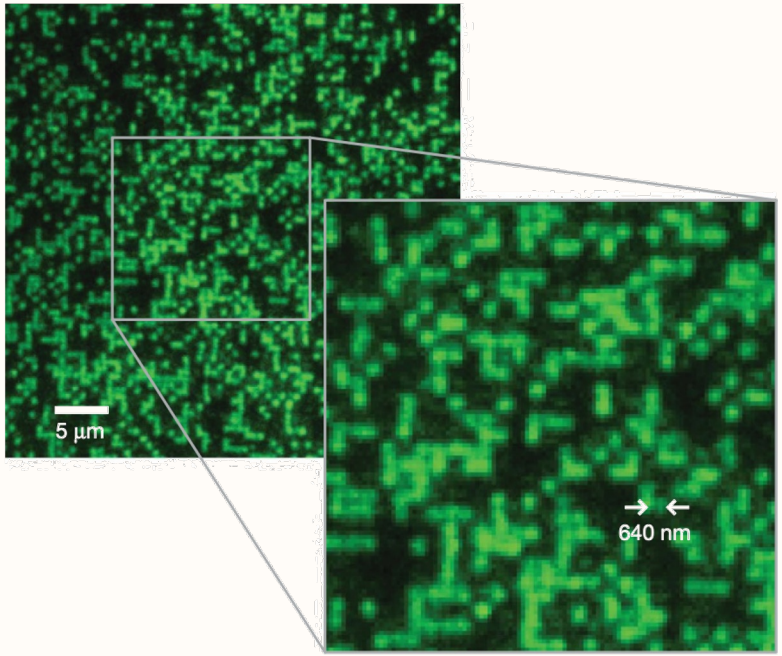


Figure 2 Absorption images of multiple matter wave interference patterns. These were obtained after suddenly releasing the atoms from an optical lattice potential with different potential depths V_0 after a time of flight of 15 ms. Values of V_0 were: **a**, 0 E_r; **b**, 3 E_r; **c**, 7 E_r; **d**, 10 E_r; **e**, 13 E_r; **f**, 14 E_r; **g**, 16 E_r; and **h**, 20 E_r.

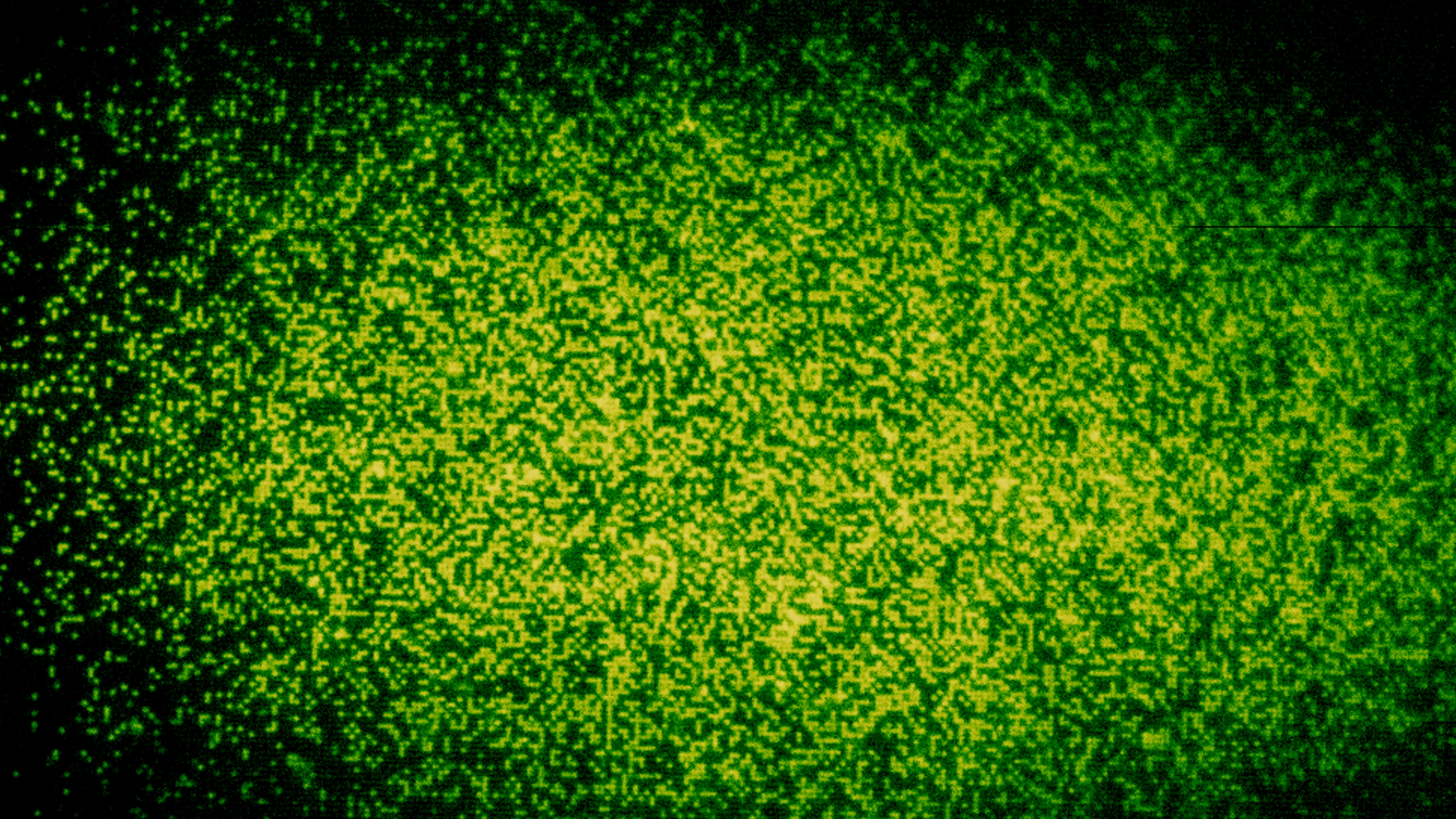
M. Greiner et al., *Nature* **415**, 39 (2002)

QUANTUM GAS MICROSCOPY

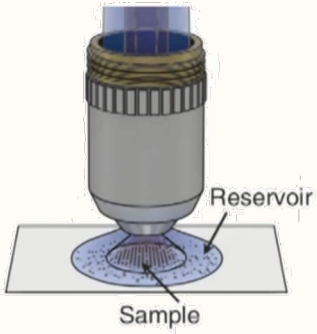


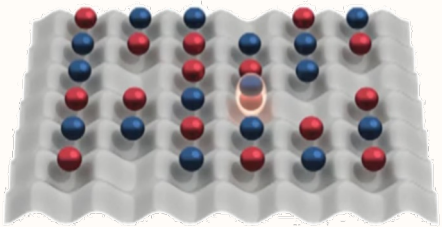


Greiner group (Harvard)
W. Bakr et al., *Science* **329**, 547 (2010)



MEASURING THE MANY-BODY STATE





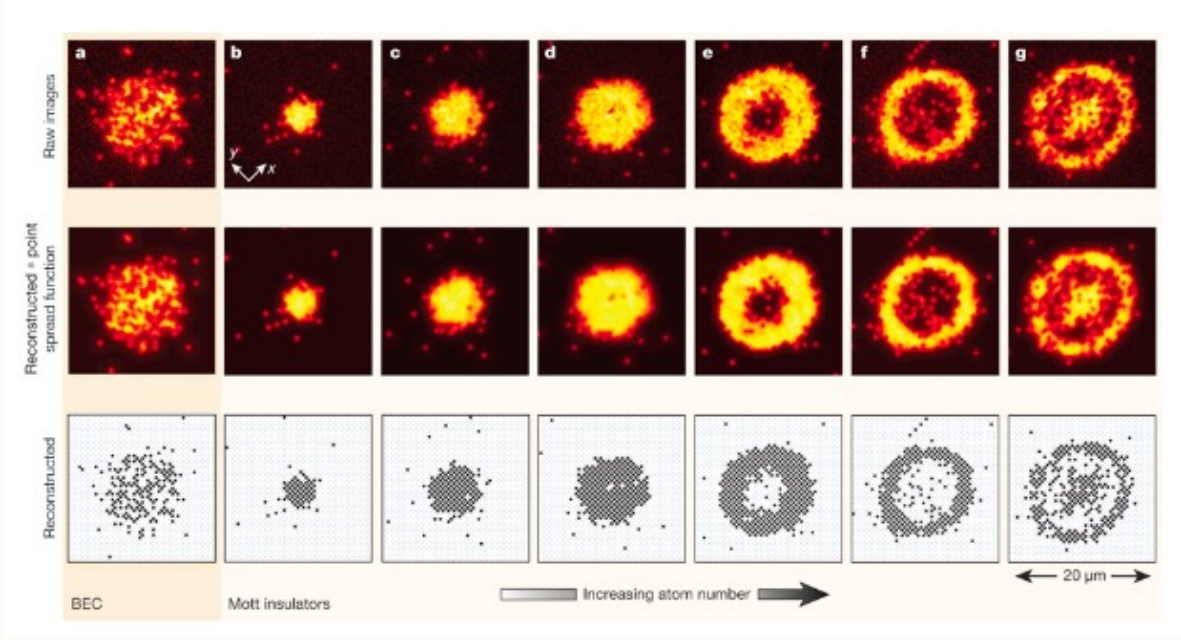
Site-resolved occupation measurement

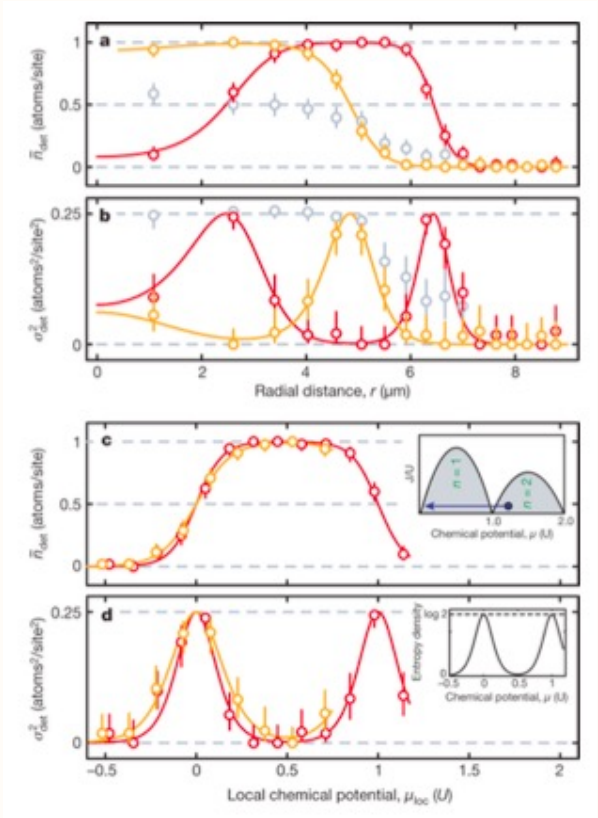
$$|\psi\rangle = |\,\rangle + |\,\rangle + |\,\rangle \cdots$$

Extendable to other observables

- All density correlations between sites/particles
- Local currents
- Entanglement entropy
- Quantum state purity
- …

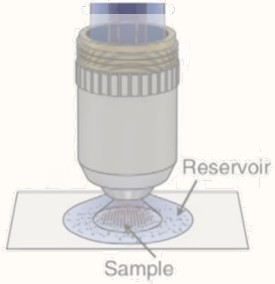
SITE-RESOLVED IMAGING



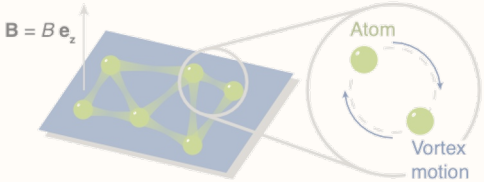


Bloch group (MPQ Munich)
J. Sherson et al., *Nature* **467**, 68 (2010)

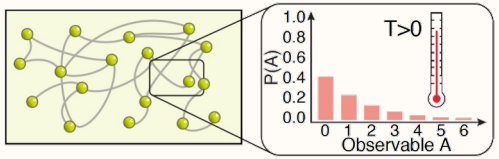
OUTLINE



Quantum gas microscopy

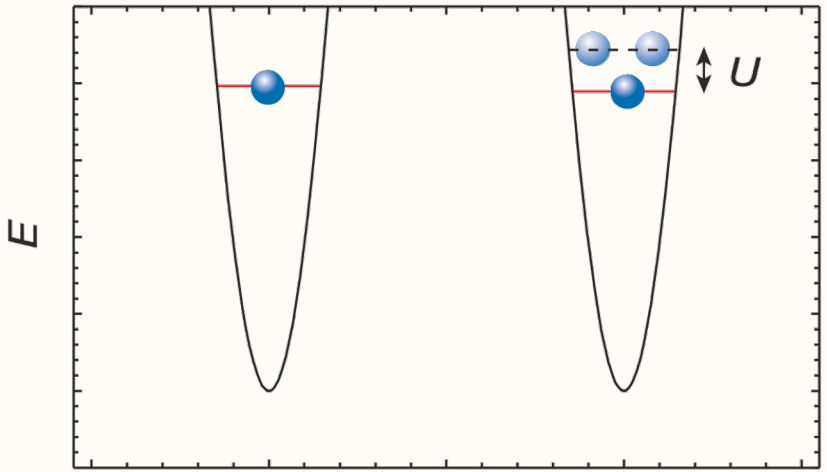


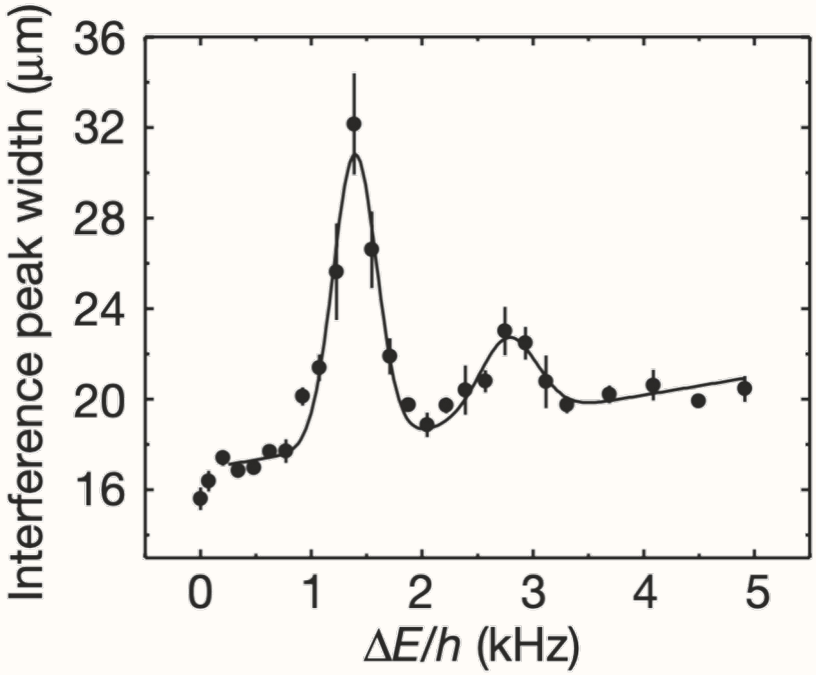
Fractional quantum Hall states



Non-equilibrium dynamics

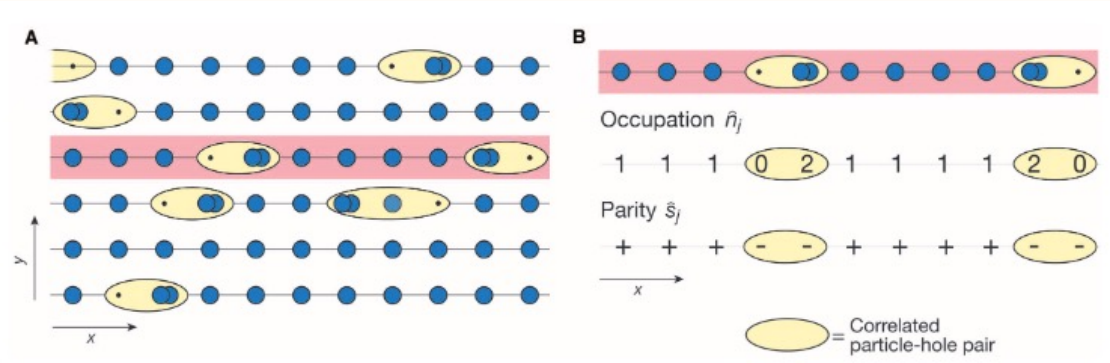
PARTICLE-HOLE PAIRS

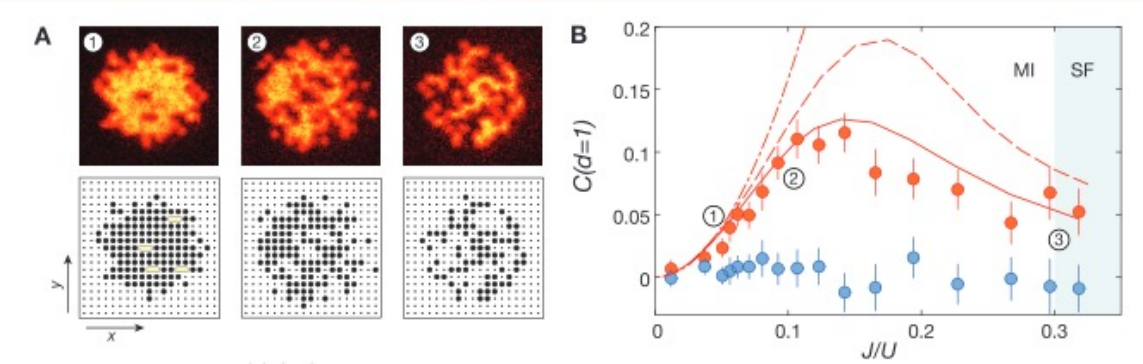




M. Greiner et al., *Nature* **415**, 39 (2002)

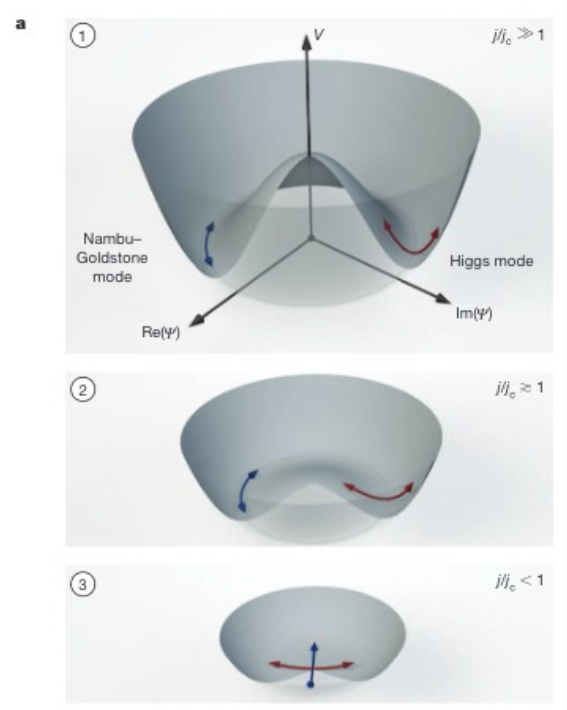
PARTICLE-HOLE PAIRS

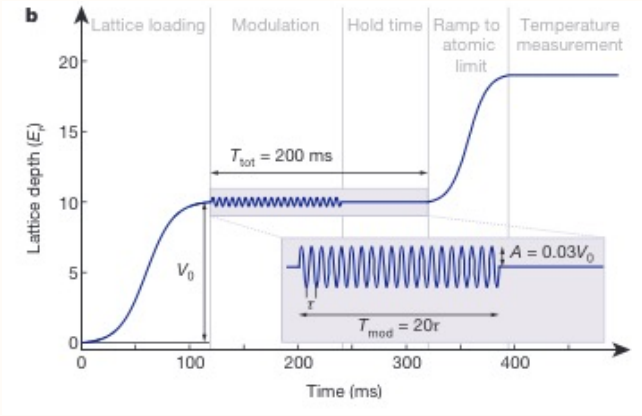


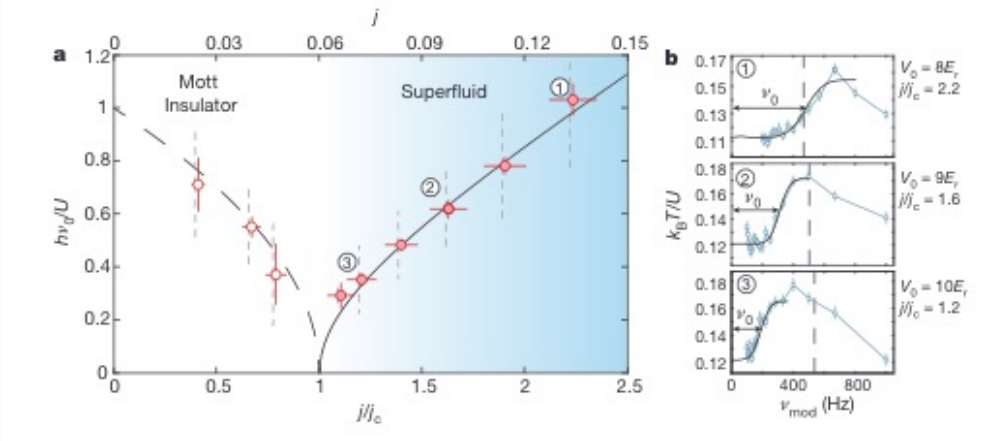


Bloch group (MPQ Munich)
M. Endres et al., *Science* **334**, 200 (2011)

HIGGS MODE







Bloch group (MPQ Munich)
M. Endres et al., *Nature* **487**, 454 (2012)

THANK YOU