Disorder-induced shift of condensation temperature for dilute trapped Bose gases

M. Timmer(*), A. Pelster(**) and R. Graham(***)

Universität Duisburg-Essen, Campus Duisburg, Fachbereich Physik
Lotharstrasse 1, 47048 Duisburg, Germany

Abstract. – We determine the leading shift of the Bose-Einstein condensation temperature
for an ultracold dilute atomic gas in a harmonic trap due to weak disorder by treating both a
Gaussian and a Lorentzian spatial correlation for the quenched disorder potential. Increasing
the correlation length from values much smaller than the geometric mean of the trap scale and
the mean particle distance to much larger values leads first to an increase of the positive shift
to a maximum at this critical length scale and then to a decrease.

The experimental realization of Bose-Einstein condensates (BECs) in magnetically trapped
dilute atomic vapors [1, 2] has renewed the theoretical interest in the phase structure of ul-
tracold Bose gases. In particular, many research groups have investigated how the transition
temperature T_c changes due to a weak local repulsive two-particle interaction $V^{(\text{int})}(\boldsymbol{x}) = 4\pi a/M \, \delta(\boldsymbol{x})$ in different trap configurations. Here a denotes the s-wave scattering length, M
the atomic mass, and we employ units with $k_B = \hbar = 1$. Theoretically most demanding
is the homogeneous case [3] where the Bose gas of particle density n resides in an infinitely
wide trap with a flat potential. There the leading shift $\Delta T_c/T_c^{(0)} = c_1 a n^{1/3}$ with respect
to the interaction-free critical temperature $T_c^{(0)} = 2\pi \left[n/\zeta(3/2) \right]^{2/3} /M$ is completely due to
long-wavelength critical fluctuations that have to be described nonperturbatively. It can be
shown that this shift follows from the classical limit of many-body theory, where only the
zero Matsubara modes of the Bose fields are included [4]. A self-consistent variational re-
summation [5] of the resulting infrared divergent series for c_1 yields the value 1.27 ± 0.11 [6]
which agrees with the Monte Carlo calculations 1.32 ± 0.02 [7,8]. In contrast to that, harmonic
trapping potentials remove the critical long-wavelength fluctuations and reduce the fraction of
atoms taking part in nonperturbative physics at the transition point. As a result, the leading
shift $\Delta T_c/T_c^{(0)} = d_1 \, a/\lambda_c^{(0)}$ in the critical temperature with respect to its noninteracting value
$T_c^{(0)} = [N\omega_{\text{g}}/\zeta(3)]^{1/3}$ is negative and can be calculated by simple perturbative methods, for

(*) E-mail: sp0088@theo-phys.uni-essen.de
(**) E-mail: axel.pelster@uni-due.de
(***) E-mail: robert.graham@uni-due.de

instance a Hartree-Fock mean-field approximation [9]. Here $\lambda_c^{(0)} = \sqrt{2\pi/[T_c^{(0)}M]}$ denotes the thermal wavelength and N the boson number, whereas $\omega_{\mathrm{g}} = (\omega_x\omega_y\omega_z)^{1/3}$ is the geometric mean of the respective trap frequencies. The analytically known constant d_1 has the numerical value $d_1 = -3.426$ which could be confirmed in a recent experiment within the error bars [10]. Note that a theoretical investigation of the crossover between both physically distinct cases of the homogeneous and the trapped Bose gas has been elaborated in ref. [11] with the help of general power law potentials. Furthermore, corrections to the leading interaction-induced shift in the critical temperature have been investigated in refs. [12–15].

Apart from the contact interaction, another interesting physical parameter for ultracold atomic vapors is frozen disorder. It can be created artificially by laser speckles [16–19], incommensurate lattices [20], or different localized atomic species [21, 22]. However, random potentials can also arise naturally via the spatial fluctuations of the electric currents generating the magnetic wire traps [23, 24]. Thus, one can raise the fundamental question how the Bose-Einstein condensation temperature is affected by weak disorder. To this end, we consider the quite commonly employed model of a Gaussian-distributed disorder potential $V(\boldsymbol{x})$ with the ensemble averages $\overline{V(\boldsymbol{x})} = 0$ and $\overline{V(\boldsymbol{x})V(\boldsymbol{x}')} = R(\boldsymbol{x} - \boldsymbol{x}')$. In particular, we are interested in correlation functions $R(\boldsymbol{x} - \boldsymbol{x}')$ which decay with a characteristic correlation length ξ. A typical example for such a correlation function is a Lorentzian distribution, which occurs in atom chips [25]; more complicated correlation functions can be designed by using laser speckles [26]. Note that the latter experiments allow to tune the corresponding correlation length ξ in a wide parameter range by changing, for instance, the distance between the scattering surface and the magneto-optical trap. So far, the disorder-induced shift of the critical temperature has only been analyzed for a homogeneous Bose gas in the limit $\xi \to 0$ of a short-range correlation $R(\boldsymbol{x}) = R\,\delta(\boldsymbol{x})$. A perturbative calculation of Lopatin and Vinokur [27], motivated by studying the superfluid motion of helium in porous media such as Vycor [28–30], yields

$$\frac{\Delta T_c}{T_c^{(0)}} = -\frac{M^2 R}{3\pi[\zeta(3/2)]^{2/3}n^{1/3}}\,. \tag{1}$$

The negative sign for the homogeneous Bose gas is compatible with the fact that disorder leads to an effective attractive interaction when the replica method is applied [31].

The purpose of this letter is to determine the leading disorder-induced shift in the critical temperature for a weakly interacting Bose gas in an external harmonic trap $U(\boldsymbol{x})$ with an additional weak disorder potential which is characterized by an arbitrary correlation function $R(\boldsymbol{x} - \boldsymbol{x}')$. According to our introduction we expect that disorder leads to an opposite sign of the shift of the critical temperature than repulsive interaction. Since disorder appears within the replica method as an effective attractive interaction [31], the particle density in the center of a harmonic trap is increased. Thus, there should be a positive disorder-induced shift for such a harmonic confinement. Moreover, we should be able to determine it reliably by a perturbative calculation. In first-order perturbation theory, the effects of both interaction and disorder are additive. Therefore, we start with the imaginary-time action of a noninteracting Bose gas in both a trap and a random potential in standard notation

$$\mathcal{A}[\psi^*,\psi] = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x\,\psi^*(\boldsymbol{x},\tau)\Big[\frac{\partial}{\partial\tau} - \frac{1}{2M}\boldsymbol{\Delta} + U(\boldsymbol{x}) + V(\boldsymbol{x}) - \mu\Big]\psi(\boldsymbol{x},\tau)\,, \tag{2}$$

and treat the disorder potential $V(\boldsymbol{x})$ perturbatively. We aim at determining the particle number $N = -\partial\Omega/\partial\mu$ from the grand-canonical free energy $\Omega = -\overline{\ln\mathcal{Z}}/\beta$, where the partition

function is determined by the functional integral

$$\mathcal{Z} = \oint \mathcal{D}\psi^* \oint \mathcal{D}\psi \, e^{-\mathcal{A}[\psi^*,\psi]} \,. \tag{3}$$

For an ideal Bose gas a semiclassical treatment of the trap potential leads to

$$\Omega^{(0)} = \frac{-1}{\beta\lambda^3} \int \mathrm{d}^3x \, \zeta_{5/2}\left(e^{\beta[\mu-U(\boldsymbol{x})]}\right) \,, \tag{4}$$

where $\zeta_a(z) = \sum_{n=1}^{\infty} z^n/n^a$ is the polylogarithmic function. Correspondingly, the first-order correction reads

$$\Omega^{(1)} = -\frac{1}{2\beta} \int_0^\beta \mathrm{d}\tau_1 \int_0^\beta \mathrm{d}\tau_2 \int \mathrm{d}^3x_1 \int \mathrm{d}^3x_2 R(\boldsymbol{x}_1 - \boldsymbol{x}_2) G^{(0)}(\boldsymbol{x}_1,\tau_1;\boldsymbol{x}_2,\tau_2) G^{(0)}(\boldsymbol{x}_2,\tau_2;\boldsymbol{x}_1,\tau_1) \,, \tag{5}$$

where the bosonic Green function has the semiclassical Fourier-Matsubara decomposition

$$G^{(0)}(\boldsymbol{x}_1,\tau_1;\boldsymbol{x}_2,\tau_2) = \frac{1}{\beta} \sum_{m=-\infty}^{\infty} \int \frac{\mathrm{d}^3k}{(2\pi)^3} e^{-i\omega_m(\tau_1-\tau_2)+i\boldsymbol{k}(\boldsymbol{x}_1-\boldsymbol{x}_2)} \, G^{(0)}\left(\omega_m,\boldsymbol{k};\frac{\boldsymbol{x}_1+\boldsymbol{x}_2}{2}\right) \tag{6}$$

with $G^{(0)}(\omega_m,\boldsymbol{k};\boldsymbol{x}) = \left[-i\omega_m + \boldsymbol{k}^2/2M + U(\boldsymbol{x}) - \mu\right]^{-1}$, $\boldsymbol{x} = (\boldsymbol{x}_1+\boldsymbol{x}_2)/2$, and $\omega_m = 2\pi m/\beta$. Evaluating in (5) the Matsubara series with the Poisson summation formula [32] and the Fourier integrals with the formula of Plemelj [33], we obtain

$$\Omega^{(1)} = \left(\frac{M}{2\pi}\right)^3 \sum_{n=1}^{\infty} \int \mathrm{d}^3x \, e^{\beta n[\mu-U(\boldsymbol{x})]} \int \mathrm{d}^3x' \, R(\boldsymbol{x}') \lim_{\epsilon \searrow 0} \mathrm{Im} \int_0^\infty \mathrm{d}\tau \, \frac{e^{-\epsilon\tau}}{[\tau(\tau+i\beta n)]^{3/2}} \times$$
$$\times \exp\left[-\frac{M\beta n\boldsymbol{x}'^2}{2\tau(\tau+i\beta n)}\right] \,. \tag{7}$$

This disorder correction to the grand-canonical free energy is now specialized for a harmonic trap $U(\boldsymbol{x})$ and for typical correlation functions $R(\boldsymbol{x}) = \int \mathrm{d}^3k/(2\pi)^3 \, e^{i\boldsymbol{k}\boldsymbol{x}} R(\boldsymbol{k})$. For instance, in case of a Gaussian $R_{\mathrm{G}}(\boldsymbol{k}) = R \, e^{-\boldsymbol{k}^2\xi^2/2}$ and a Lorentzian $R_{\mathrm{L}}(\boldsymbol{k}) = R/(1+\boldsymbol{k}^2\xi^2)$, we find

$$\Omega_{\mathrm{G}}^{(1)} = -\frac{\beta R}{2(2\pi)^{3/2}(\beta\omega_{\mathrm{g}})^3\xi} \sum_{n=1}^{\infty} \frac{e^{n\beta\mu}}{n^2\left(\xi^2 + \frac{\beta n}{4M}\right)} \tag{8}$$

and, correspondingly,

$$\Omega_{\mathrm{L}}^{(1)} = -\frac{\pi\sqrt{M\beta}R}{2(2\pi)^{3/2}\xi^2(\beta\omega_{\mathrm{g}})^3} \sum_{n=1}^{\infty} \frac{e^{\beta n\mu}}{n^{5/2}} \exp\left[\frac{\beta n}{8M\xi^2}\right] \left[1 - \Phi\left(\sqrt{\frac{\beta n}{8M\xi^2}}\right)\right] \,, \tag{9}$$

where we have used for the latter the probability integral $\Phi(z) = 2/\sqrt{\pi} \int_0^z \mathrm{d}t \, e^{-t^2}$.

As follows from the theory of critical phenomena [34,35], the phase transition to a Bose-Einstein condensate occurs when the spatio-temporal integral over the correlation function

$$G(\boldsymbol{x}_1,\tau_1;\boldsymbol{x}_2,\tau_2) = \overline{\frac{1}{\mathcal{Z}} \oint \mathcal{D}\psi^* \oint \mathcal{D}\psi \, \psi(\boldsymbol{x}_1,\tau_1)\psi^*(\boldsymbol{x}_2,\tau_2) \, e^{-\mathcal{A}[\psi^*,\psi]}} \tag{10}$$

diverges, *i.e.* $G(\omega_m, \boldsymbol{k}; \boldsymbol{x})$ becomes infinite for $\omega_m = 0$, $\boldsymbol{k} = \boldsymbol{0}$, and some $\boldsymbol{x}$. This criticality criterion is equivalent to the vanishing of its inverse

$$G^{-1}(0, \boldsymbol{0}; \boldsymbol{x}) = G^{(0)\,-1}(0, \boldsymbol{0}; \boldsymbol{x}) - \Sigma(0, \boldsymbol{0}; \boldsymbol{x}), \tag{11}$$

where the first-order perturbative contribution to the self-energy is given by

$$\Sigma^{(1)}(\boldsymbol{x}_1, \tau_1; \boldsymbol{x}_2, \tau_2) = R(\boldsymbol{x}_1 - \boldsymbol{x}_2) G^{(0)}(\boldsymbol{x}_1, \tau_1; \boldsymbol{x}_2, \tau_2). \tag{12}$$

Therefore, we define the critical chemical potential μ_c from the condition

$$\underset{\boldsymbol{x}}{\text{Min}} \; G^{-1}(0, \boldsymbol{0}; \boldsymbol{x}) = 0. \tag{13}$$

Up to first order in the disorder we obtain a result which does not depend on the trap frequencies:

$$\mu_c^{(1)} = -2M \int \mathrm{d}^3 x \, \frac{R(\boldsymbol{x})}{|\boldsymbol{x}|}. \tag{14}$$

We conclude from (14) that Bose-Einstein condensation occurs with disorder earlier than in the clean case. Note that the critical chemical potential for the onset of Bose-Einstein condensation follows from condition (13) also in other physical situations, for instance, when anisotropic traps are used for rotating Bose gases [36] or when dipolar gases are considered where the two-particle interaction contains both a short-range, isotropic contact potential and a long-range, anisotropic dipole-dipole interaction [37, 38].

Now we combine our previous results and determine the disorder-induced shift in the critical temperature. To this end, we evaluate $N = N(\mu)$ in the limit $\mu \uparrow \mu_c$ by taking into account the Robinson formula [39] for the polylogarithmic function in (4). For the Gaussian correlation we find

$$\frac{\Delta T_{c,\mathrm{G}}}{T_c^{(0)}} = \frac{8\tilde{R}\tilde{\xi}}{3(2\pi)^{3/2}\zeta(3)} \sum_{k=1}^{\infty} \frac{1}{k^2 \left[k + 4N^{1/3}\tilde{\xi}^2/[\zeta(3)]^{1/3}\right]}, \tag{15}$$

whereas we obtain for the Lorentzian correlation

$$\frac{\Delta T_{c,\mathrm{L}}}{T_c^{(0)}} = \frac{\tilde{R}}{6\pi[\zeta(3)]^{2/3}\tilde{\xi}N^{1/3}} \sum_{k=1}^{\infty} \left\{ \frac{1}{k^2} - \frac{[\zeta(3)]^{1/6}\sqrt{\pi}}{\sqrt{8}\tilde{\xi}N^{1/6}k^{3/2}} \times \right.$$
$$\left. \times \exp\left[\frac{[\zeta(3)]^{1/3}k}{8\tilde{\xi}^2 N^{1/3}}\right] \left[1 - \Phi\left(\sqrt{\frac{[\zeta(3)]^{1/3}k}{8\tilde{\xi}^2 N^{1/3}}}\right)\right] \right\}. \tag{16}$$

Here we have introduced dimensionless units for the correlation length $\tilde{\xi} = \xi/l_{\mathrm{os}}$ as well as for the disorder strength $\tilde{R} = M^2 l_{\mathrm{os}} R$, where $l_{\mathrm{os}} = 1/\sqrt{M\omega_{\mathrm{g}}}$ denotes the oscillator length. Note that both disorder-induced shifts (15) and (16) have similar asymptotics in the limit $\xi \to 0$

$$\frac{\Delta T_{c,\mathrm{G}}}{T_c^{(0)}} \approx \frac{8\tilde{R}\tilde{\xi}}{3(2\pi)^{3/2}}, \qquad \frac{\Delta T_{c,\mathrm{L}}}{T_c^{(0)}} \approx \frac{2\tilde{R}\tilde{\xi}}{3\pi} \tag{17}$$

and in the limit $\xi \to \infty$

$$\frac{\Delta T_{c,\mathrm{G}}}{T_c^{(0)}} \approx \frac{2\zeta(2)\tilde{R}}{3(2\pi)^{3/2}[\zeta(3)]^{2/3}\tilde{\xi}N^{1/3}}, \qquad \frac{\Delta T_{c,\mathrm{L}}}{T_c^{(0)}} \approx \frac{\zeta(2)\tilde{R}}{6\pi[\zeta(3)]^{2/3}\tilde{\xi}N^{1/3}}. \tag{18}$$

$$\frac{\Delta T_c N^{1/6}}{T_c^{(0)} \tilde{R}}$$
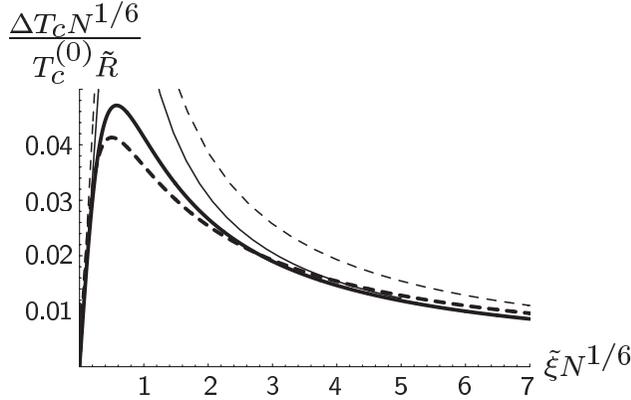
$\tilde{\xi} N^{1/6}$

Fig. 1 – Disorder-induced shifts (15) and (16) of the critical temperature ΔT_c with respect to the interaction-free critical temperature $T_c^{(0)}$ for the Gaussian (solid) and the Lorentzian (dashed) correlation function together with their asymptotics (17) and (18) as a function of the dimensionless correlation length.

Figure 1 shows how the shift results (15) and (16) vary with the dimensionless correlation length $\tilde{\xi}$. Whereas for small $\tilde{\xi}$ the oscillator length l_{os} dominates, the average particle distance $l_{\mathrm{p}} = l_{\mathrm{os}}/N^{1/3}$ becomes important for large $\tilde{\xi}$. Both length scales meet at $\sqrt{l_{\mathrm{os}} l_{\mathrm{p}}} = l_{\mathrm{os}}/N^{1/6}$, i.e. $\tilde{\xi} N^{1/6} = 1$, which is roughly where the maximal shift occurs. This result will certainly help to design an experiment which aims at detecting a disorder-induced shift of the critical temperature.

Furthermore, it is clear from (15), (16), and fig. 1 that the shifts for Gaussian or Lorentzian spatial correlations are qualitatively and even semi-quantitatively the same. This finding justifies to apply our results also to other correlation functions, which decay with a characteristic correlation length ξ as, for instance, the laser speckle correlations in the Florence experiment [16, 18]. There about $N = 3 \cdot 10^5$ atoms of the ^{85}Rb isotope reside in an anisotropic harmonic trap with the geometric mean frequency $\omega_{\mathrm{g}} = 2\pi \cdot 40$ Hz, so that its oscillator
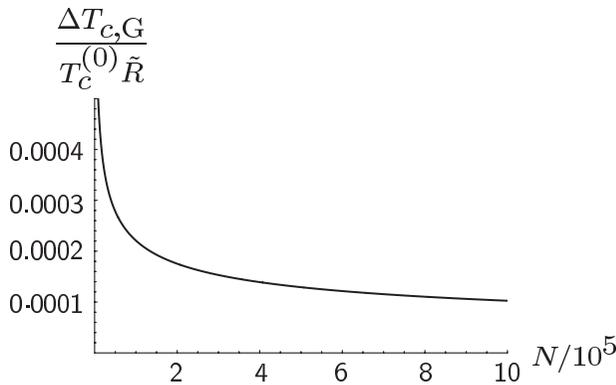

$$\frac{\Delta T_{c,\mathrm{G}}}{T_c^{(0)} \tilde{R}}$$

$N/10^5$

Fig. 2 – Normalized shift of the critical temperature $\Delta T_{c,\mathrm{G}}/(T_c^{(0)} \tilde{R})$ of a dilute ^{85}Rb gas in an anisotropic harmonic trap with geometric mean frequency $\omega_{\mathrm{g}} = 2\pi \cdot 40$ Hz and a Gaussian disorder correlation with correlation length $\xi = 10\,\mu$m *vs.* particle number N.

length is $l_{\text{os}} = 1.72\,\mu$m. The disorder potential is characterized by a length scale $\xi = 10\,\mu$m, *i.e.* $\tilde{\xi} = 5.88$, and a strength R which is measured in units of a frequency ω. As R has the physical units $(\text{energy})^2(\text{length})^3$, the dimensionless disorder strength can be estimated by $\tilde{R} = (\omega/\omega_{\text{g}})^2\tilde{\xi}^3$. A weak disorder potential, which only slightly perturbes the condensate in the Florence experiment, corresponds to the frequency $\omega = 2\pi \cdot 50$ Hz, yielding $\tilde{R} = 200$. In fig. 2 the normalized shift of the critical temperature $\Delta T_c/(T_c^{(0)}\tilde{R})$ is plotted as a function of the particle number N for a Gaussian correlation, so we conclude that the disorder-induced shift in the critical temperature is in this case of the order of 3%. With this it is about the same size as the interaction-induced shift [9] and should thus be measurable [10]. Furthermore, we read off from fig. 2 that this shift increases with a decreasing number of particles in the Bose gas.

In this letter we have determined the disorder-induced shift of the critical temperature for a Bose gas in a harmonic trap. By treating the analytically solvable cases of a Gaussian and a Lorentzian correlation function, we have estimated that this disorder-induced shift should be measurable in laser speckle experiments. Note that applying our general perturbative results (5) and (14) to a homogeneous Bose gas and to a vanishing correlation length $\xi = 0$ reproduces the seminal Lopatin-Vinokur shift (1). However, in this case the interaction-induced shift of the critical temperature suggests that additional nonperturbative contributions to the disorder-induced shift will certainly occur. First indications for this conjecture are provided by a recent momentum shell renormalization group calculation [40].

$$* * *$$

REFERENCES

[1] Anderson M. H., Ensher J. R., Matthews M. R., Wieman C. E. and Cornell E. A., *Science*, **269** (1995) 198.
[2] Davis K. B., Mewes M.-O., Andrews M. R., van Druten N. J., Durfee D. S., Kurn D. M. and Ketterle W., *Phys. Rev. Lett.*, **75** (1995) 3969.
[3] Andersen J. O., *Rev. Mod. Phys.*, **76** (2004) 599.
[4] Baym G., Blaizot J. P., Holzmann M., Laloë F. and Vautherin D., *Phys. Rev. Lett.*, **83** (1999) 1703.
[5] Kleinert H., *Mod. Phys. Lett. B*, **17** (2003) 1011.
[6] Kastening B., *Phys. Rev. A*, **69** (2004) 043613.
[7] Kashurnikov V. A., Prokof'ev N. V. and Svistunov B. V., *Phys. Rev. Lett.*, **87** (2001) 120402.
[8] Arnold P. and Moore G., *Phys. Rev. Lett.*, **87** (2001) 120401.
[9] Giorgini S., Pitaevskii L. P. and Stringari S., *Phys. Rev. A*, **54** (1996) R4633.
[10] Gerbier F., Thywissen J. H., Richard S., Hugbart M., Bouyer P. and Aspect A., *Phys. Rev. Lett.*, **92** (2004) 030405.
[11] Zobay O., Metikas G. and Kleinert H., *Phys. Rev. A*, **71** (2005) 043614
[12] Arnold P., Moore G. and Tomášik B., *Phys. Rev. A*, **65** (2002) 013606.
[13] Arnold P. and Tomášik B., *Phys. Rev. A*, **64** (2001) 053609.
[14] Davis M. J. and Blakie P. B., *Phys. Rev. Lett.*, **96** (2006) 060404.
[15] Kao Y.-M. and Jiang T. F., *Phys. Rev. A*, **73** (2006) 043604.
[16] Lye J. E., Fallani L., Modugno M., Wiersma D. S., Fort C. and Inguscio M., *Phys. Rev. Lett.*, **95** (2005) 070401.
[17] Clément D., Varón A. F., Hugbart M., Retter J. A., Bouyer P., Sanchez-Palencia L., Gangardt D. M., Shlyapnikov G. V. and Aspect A., *Phys. Rev. Lett.*, **95** (2005) 170409.

[18] FORT C., FALLANI L., GUARRERA V., LYE J. E., MODUGNO M., WIERSMA D. S. and INGUSCIO M., *Phys. Rev. Lett.*, **95** (2005) 170410.

[19] SCHULTE T., DRENKELFORTH S., KRUSE J., ERTMER W., ARLT J., SACHA K., ZAKRZEWSKI J. and LEWENSTEIN M., *Phys. Rev. Lett.*, **95** (2005) 170411.

[20] DAMSKI B., ZAKRZEWSKI J., SANTOS L., ZOLLER P. and LEWENSTEIN M., *Phys. Rev. Lett.*, **91** (2003) 080403.

[21] GAVISH U. and CASTIN Y., *Phys. Rev. Lett.*, **95** (2005) 020401.

[22] OSPELKAUS S., OSPELKAUS C., WILLE O., SUCCO M., ERNST P., SENGSTOCK K. and BONGS K., *Phys. Rev. Lett.*, **96** (2006) 180403.

[23] WANG D.-W., LUKIN M. D. and DEMLER E., *Phys. Rev. Lett.*, **92** (2004) 076802.

[24] SCHUMM T., ESTEVE J., FIGL C., TREBBIA J.-B., AUSSIBAL C., NGUYEN H., MAILLY D., BOUCHOULE I., WESTBROOK C. I. and ASPECT A., *Eur. Phys. J. D*, **32** (2005) 171.

[25] FOLMAN R., KRÜGER P., SCHMIEDMAYER J., DENSCHLAG J. and HENKEL C., *Adv. At. Mol. Opt. Phys.*, **48** (2002) 263.

[26] DAINTY J. C. (Editor), *Laser Speckle and Related Phenomena* (Springer, Berlin) 1975.

[27] LOPATIN A. V. and VINOKUR V. M., *Phys. Rev. Lett.*, **88** (2002) 235503.

[28] CROOKER B. C., HEBRAL B., SMITH E. N., TAKANO Y. and REPPY J. D., *Phys. Rev. Lett.*, **51** (1983) 666.

[29] CHAN M. H. W., BLUM K. I., MURPHY S. Q., WONG G. K. S. and REPPY J. D., *Phys. Rev. Lett.*, **61** (1988) 1950.

[30] HUANG K. and MENG H. F., *Phys. Rev. Lett.*, **69** (1992) 644.

[31] GRAHAM R. and PELSTER A., eprint: cond-mat/0508306.

[32] KLEINERT H., *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, fourth extended edition (World Scientific, Singapore) 2006.

[33] HEITLER W., *The Quantum Theory of Radiation*, third edition (Oxford University Press, Oxford) 1970.

[34] ZINN-JUSTIN J., *Quantum Field Theory and Critical Phenomena*, 3rd edition (Oxford University Press, Oxford) 1996.

[35] KLEINERT H. and SCHULTE-FROHLINDE V., *Critical Properties of Φ^4-Theories* (World Scientific, Singapore) 2001.

[36] KLING S. and PELSTER A., eprint: cond-mat/0604162.

[37] GLAUM K., PELSTER A., KLEINERT H. and PFAU T., eprint: cond-mat/0606569.

[38] GLAUM K. and PELSTER A., eprint: cond-mat/0609374.

[39] ROBINSON J. E., *Phys. Rev.*, **83** (1951) 678.

[40] ZOBAY O., *Phys. Rev. A*, **73** (2006) 023616.