SPINOR FERMI GASES

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At first, we consider an ideal gas of harmonically trapped fermions with total angular momentum $F = 3/2$ and calculate, *e. g.*, the temperature dependence of the heat capacity for a fixed magnetization. Afterwards, the isotropic short-range contact-interaction is treated perturbatively and its influence on the ground-state energy is worked out. Such spinor Fermi gases are important, for example, in the context of a ^{52}Cr-^{53}Cr boson-fermion mixture.[1]

1. Introduction

Since the first observations of Bose-Einstein condensation (BEC) in ultracold dilute gases of alkali metals,[2,3] the interest in fermionic gases has also increased. Superfluidity in fermionic species has been shown to be deeply related to the BEC of Cooper pairs, as originally introduced in the context of the Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity. The recent observation of the BCS-BEC-crossover[4] in ^{6}Li, as theoretically predicted,[5] has emphasised how important the underlying fermionic degrees of freedom are for the physics of ultracold degenerate quantum gases.

Here, we are mainly interested in the spinorial character of Fermi gases with total angular momentum, denoted by F, larger than $1/2$. From now on, we assume that fermionic atoms with large F are cooled and trapped in the harmonic potential of an optical trap, where the spinorial character becomes particularly enhanced. One could choose fermionic isotopes of ytterbium, potassium, chromium, etc. Let us consider ^{53}Cr for definiteness. We remark that its bosonic counterpartner ^{52}Cr has been Bose-condensed[6] and that

its large magnetic moment of $6\mu_B$, where μ_B denotes the Bohr magneton, makes it an interesting system in itself. Consider the ground-state of ^{53}Cr. The nuclear spin $I = 3/2$ added to the electronic spin $S = 3$ renders it four-fold degenerate with F ranging from $F = |I - S| = 3/2$ to $F = I + S = 9/2$. This suggests to investigate, at first, ideal Fermi gases with large F.

2. Ideal Fermi Gases

The euclidean action $\mathcal{A}^{(0)}$ of a non-interacting system is given by

$$\mathcal{A}^{(0)} = \int_0^{\hbar\beta} \mathrm{d}\tau \int \mathrm{d}^D x \, \Psi^\dagger \left[\hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2 \nabla^2}{2M} + U(\mathbf{x}) - \mu - \eta F_z \right] \Psi. \qquad (1)$$

Here, $\Psi(\mathbf{x}, \tau)$ is a grassmannian spinor field with the ad-joint $\Psi^\dagger(\mathbf{x}, \tau) = \left(\psi^*_{+F}(\mathbf{x}, \tau), \cdots, \psi^*_{-F}(\mathbf{x}, \tau)\right)$ and F_z is the z-component of the spin matrix. Both the chemical potential μ and the magneto-chemical one η are functions of the temperature and represent Lagrange parameters for fixing the particle number and the magnetization in z-direction, respectively.

The grand-canonical partition function $\mathcal{Z}^{(0)}$ is defined by

$$\mathcal{Z}^{(0)} = \oint \mathcal{D}\Psi^\dagger \mathcal{D}\Psi \; e^{-\mathcal{A}^{(0)}/\hbar}, \qquad (2)$$

where the functional integration runs over all spinor fields, which are anti-periodic in imaginary time. Treating the harmonic potential $U(\mathbf{x}) = M \sum_{i=1}^{3} \omega_i^2 x_i^2/2$ semi-classically, the grand-canonical free energy $\mathcal{F}^{(0)} = -(\log \mathcal{Z}^{(0)})/\beta$ is given by

$$\mathcal{F}^{(0)} = \frac{-1}{\beta(\beta\bar{\omega}\hbar)^3} H_0^4(z, \alpha). \qquad (3)$$

Here, the function $H_n^k(z, \alpha) = \sum_{f=-F}^{+F} \frac{f^k}{\Gamma(n)} \int_0^\infty dy \frac{y^{n-1}}{1+\exp(y)(z\alpha^f)^{-1}}$ is introduced with z (α) being the fugacity (magnetic fugacity). The geometric mean of the trap frequencies is denoted by $\overline{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$.

From Eq. (3) follow all thermodynamical properties of the system. For example, the total particle number $N = -\partial\mathcal{F}/\partial\mu$ and the magnetization $\mathcal{M} = -\partial\mathcal{F}/\partial\eta$ are given by

$$N = \frac{1}{(\beta\bar{\omega}\hbar)^3} H_3^0(z, \alpha), \qquad \mathcal{M} = \frac{1}{(\beta\bar{\omega}\hbar)^3} H_3^1(z, \alpha). \qquad (4)$$

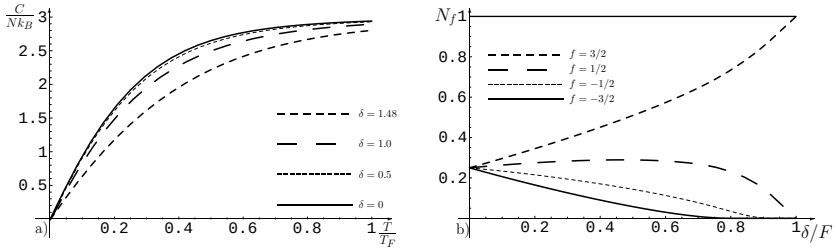


Fig. 1. Spinor Fermi gas with $F = 3/2$: a) Heat capacity versus temperature for some values of δ. b) Occupation numbers against δ at $T = 0$.

A similar expression can be derived for the heat capacity

$$\frac{C}{Nk_B} = 12\frac{H_4^0(z,\alpha)}{H_3^0(z,\alpha)} - 9\frac{H_3^1(z,\alpha)H_2^1(z,\alpha) - H_3^0(z,\alpha)H_2^2(z,\alpha)}{H_2^1(z,\alpha)^2 - H_2^0(z,\alpha)H_2^2(z,\alpha)} + 9\delta\frac{H_3^1(z,\alpha)H_2^0(z,\alpha) - H_3^0(z,\alpha)H_2^1(z,\alpha)}{H_2^1(z,\alpha)^2 - H_2^0(z,\alpha)H_2^2(z,\alpha)}, \tag{5}$$

where $\delta = \mathcal{M}/N$ characterises the magnetization per particle.

In order to evaluate thermodynamical quantities for given F and δ, one has to solve Eqs. (4) for z and α. Some examples are presented in Fig. 1 for $F = 3/2$. Part a) shows the heat capacity (5) against temperature in units of the Fermi temperature $T_F \equiv E_F/k_B$, where $E_F = \hbar\overline{\omega}(3N/2)^{1/3}$ is the Fermi energy. Part b) depicts the occupation numbers for $T = 0$ against δ.

3. Weakly Interacting Fermi Gases at $T = 0$

The energy differences between the different F substates in the ground-state of ^{53}Cr is much larger[1] than the expected trapping frequencies. This, combined with the diluteness of the system, allows to describe the low-energy dynamics of the system through a pairwise contact interaction which is rotationally invariant in the hyperfine spin space and preserves the hyperfine spin of the individual atoms. Such an interaction is described through[7]

$$V_{\imath\jmath\imath'\jmath'}^{(\mathrm{contact})}(\mathbf{x},\mathbf{x}') = \delta(\mathbf{x}-\mathbf{x}')\left[c_0\ (\mathbf{F}_{\imath\jmath}\cdot\mathbf{F}_{\imath'\jmath'})^0 + c_1\ (\mathbf{F}_{\imath\jmath}\cdot\mathbf{F}_{\imath'\jmath'})^1\right], \tag{6}$$

where the coefficients c_0 and c_1 depend only on a_0 and a_2, the scattering lengths in the $|\mathbf{F}+\mathbf{F}'| = 0, 2$-channels. The ground-state energy, up to first order in (6) with $c_0' = 5a_2 - a_0$ and $c_1' = (a_2 - a_0)$, reads

$$\frac{E}{NE_F} = \frac{3}{4} + 7\frac{N^{1/6}}{\sqrt{2\pi}}\left(\frac{3}{2}\right)^{7/6}\left\{\sum_{f,f'\neq f} A(y_<,y_>)\left[\frac{c_0'}{4\overline{a}_{\mathrm{osc}}} + f(f'+1)\frac{c_1'}{3\overline{a}_{\mathrm{osc}}}\right] + \frac{c_1'}{6\overline{a}_{\mathrm{osc}}}\sum_{f=-3/2}^{3/2} \times \left[\left(\frac{3}{2}-f\right)\left(\frac{5}{2}+f\right)A(y_f,y_{f+1}) + \left(\frac{3}{2}+f\right)\left(\frac{5}{2}-f\right)A(y_{f-1},y_f)\right]\right\}, \tag{7}$$

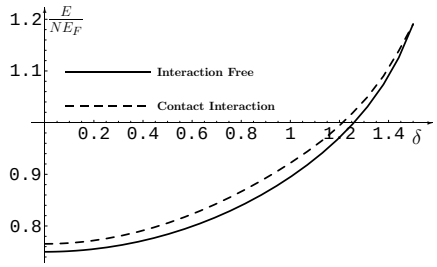


Fig. 2. Spinor Fermi gas with $F = 3/2$: Ground-state energy for $\overline{\omega} \approx 2\pi \times 100$ Hz, $a_0 \approx a_2 = 10^{-9}$m, and $N \approx 10^5$.

where $A\left(y_<, y_>\right) = {}_2F_1\left(-\frac{3}{2}, \frac{3}{2}; 4; \frac{y_<}{y_>}\right) \frac{y_<^3}{\Gamma(4)} \frac{y_>^{3/2}}{\Gamma(5/2)} \Theta(y_<)$ and $y_<$ ($y_>$) is the smallest (largest) of $y_f = \mu + f\eta$ and $y_{f'} = \mu + f'\eta$. This result for the ground-state energy is depicted in Fig. 2 as a function of δ.

4. Outlook

We have studied some properties of the weakly interacting spinor Fermi gas. Future studies are necessary to understand, *e. g.*, multi-channel superfluidity,[8] the role of the dipole-dipole interaction,[9] and their combination.

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