DENSITY AND STABILITY IN ULTRACOLD DILUTE BOSON-FERMION MIXTURES

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We analyze in detail recent experiments on ultracold dilute ^{87}Rb–^{40}K mixtures in Hamburg and in Florence within a mean-field theory. To this end we determine how the stationary bosonic and fermionic density profiles in this mixture depend in the Thomas-Fermi limit on the respective particle numbers. Furthermore, we investigate how the observed stability of the Bose-Fermi mixture with respect to collapse is crucially related to the value of the interspecies s-wave scattering length.

Keywords: Bose–Fermi-mixture; density; stability.

1. Introduction

Six years after the first experimental achievement of Bose-Einstein condensation (BEC) of trapped atomic gases in 1995 fermionic atomic gases were brought together with bosonic atoms to quantum degeneracy in a ^{7}Li–^{6}Li mixture,[1,2] ^{23}Na–^{6}Li mixture,[3] and ^{87}Rb–^{40}K mixture.[4] In such mixtures one investigates, in particular, how the two-particle interaction influences the system properties. Depending on the nature of the interspecies interaction, a repulsion between bosons and fermions tends to a demixing in order to minimize the overlapping region,[5] whereas in the case of an attraction the mixture can collapse as long as the particle numbers are sufficiently large.[6,7] Ultracold trapped boson-fermion mixtures were investigated with respect to a demixing of the components[8–10] and to a collapse due to the interspecies attraction.[9–13] Furthermore, the time-dependent dynamics of the

collapse[14] and finite-temperature effects on the stability in a boson-fermion mixture were also studied.[15]

Our theoretical investigation is based on the experiments with a ^{87}Rb–^{40}K boson-fermion mixture in a harmonic trap, which were performed in Hamburg[6] and in Florence.[4,7] The parameters of both experiments are summarized in Ref.[16] There our attention is mainly focuesed on the value of the interspecies s-wave scattering length a_{BF} since this parameter is of great importance for the system, especially for the stability of the mixture against collapsing. It turns out that different determination methods for a_{BF} lead, surprisingly, to incompatible values in the literature.[16] Thus, further investigations in this field are needed.

2. Density profiles

A boson-fermion mixture at zero temperature, where all bosons are condensed in the single-particle ground state and the fermions occupy every state below the Fermi energy, is described by the stationary Gross-Pitaevskii equation[16]

$$\left[-\frac{\hbar^2}{2m_B}\Delta + V_B(\mathbf{x}) - \mu_B + g_{BB}|\Psi_B(\mathbf{x})|^2 + g_{BF}\, n_F(\mathbf{x})\right]\Psi_B(\mathbf{x}) = 0 \quad (1)$$

with the condensate wave function $\Psi_B(\mathbf{x})$ and the fermionic particle density

$$n_F(\mathbf{x}) = \kappa\,\Theta\left(\mu_F - V_F(\mathbf{x}) - g_{BF}\, n_B(\mathbf{x})\right)\left[\mu_F - V_F(\mathbf{x}) - g_{BF}\, n_B(\mathbf{x})\right]^{3/2}, \quad (2)$$

which modulates the condensate density $n_B(\mathbf{x}) = |\Psi_B(\mathbf{x})|^2$ and vice versa. Here $V_i(\mathbf{x})$ and μ_i $(i = B, F)$ denote the external trap potential and the chemical potential for bosons and fermions, respectively, g_{BB} describes the strength of the contact interaction between two bosons and g_{BF} stands for the conrresponding one between a boson and a fermion. Assuming that the potential and interaction energy are larger than the kinetic energy, one can use the Thomas-Fermi approximation, where the kinetic term in the Gross-Pitaevskii equation (1) is neglected, so that the latter reduces to an algebraic equation with respect to the bosonic particle density $n_B(\mathbf{x})$:[17]

$$V_B(\mathbf{x}) - \mu_B + g_{BB}\, n_B(\mathbf{x}) + g_{BF}\, n_F(\mathbf{x}) = 0. \quad (3)$$

The bosonic and the fermionic density distribution solving this equation is plotted in Fig. 1 for the parameters of the Hamburg experiment.[6]

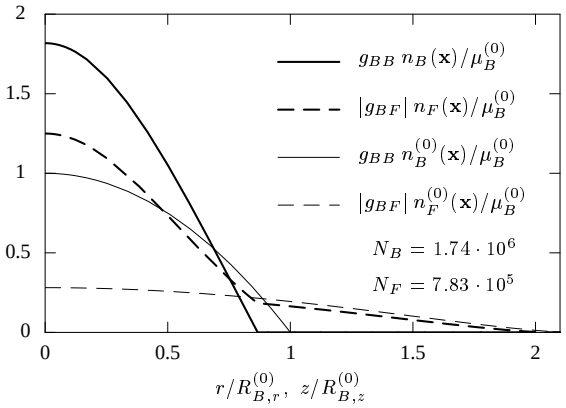


Fig. 1. Comparison of the dimensionless particle densities for bosons and for fermions between mixtures with interspecies interaction (thick lines) and without interspecies interaction (thin lines). The densities are plotted versus the coordinates r at the plane $z = 0$ and versus z at the line $r = 0$, respectively, in units of the Thomas-Fermi radii of the noninteracting BEC for an example of typical particle numbers N_B, N_F of the Hamburg experiment.[6]

3. Stability against collapse

In order to determine the border between stability and instability we extremized the grand-canonical free energy[16]

$$\mathcal{F} = \int d^3x \left\{ \Psi_B^*(\mathbf{x}) \left[-\frac{\hbar^2}{2m_B}\Delta + V_B(\mathbf{x}) - \mu_B + \frac{g_{BB}}{2} |\Psi_B(\mathbf{x})|^2 \right] \Psi_B(\mathbf{x}) - \frac{2}{5}\kappa\, \Theta(\tilde{\mu}_F(\mathbf{x}))\, \tilde{\mu}_F^{5/2}(\mathbf{x}) \right\} \quad (4)$$

of the mixture by varying the widths $\alpha\, L_{B,k}$ in the test function

$$\Psi_B(\mathbf{x}) = \sqrt{\frac{N_B \lambda^{1/2}}{\pi^{3/2}\alpha^3 L_{B,r}^3}}\, \exp\left\{ -\frac{r^2 + \lambda z^2}{2\alpha^2 L_{B,r}^2} \right\}, \qquad \lambda = (\omega_z/\omega_r)^2, \quad (5)$$

which has the form of the ground-state wave function of a three-dimensional anisotropic harmonic oscillator. Here the variational parameter α has to fulfill the conditions

$$N_B = N_{B\mathrm{crit}} \qquad \Leftrightarrow \qquad \left.\frac{d\mathcal{F}(\alpha)}{d\alpha}\right|_{\alpha=\alpha_{\mathrm{crit}}} = \left.\frac{d^2\mathcal{F}(\alpha)}{d\alpha^2}\right|_{\alpha=\alpha_{\mathrm{crit}}} = 0, \quad (6)$$

which yield the critical number of bosons N_B and the corresponding one for fermions N_F by integrating out the fermionic particle density (2). Comparing them with the data of the Hamburg experiment allows us to fit the s-wave scattering length to $a_{BF} = (-16.82 \pm 030)$ as shown in Fig. 2.

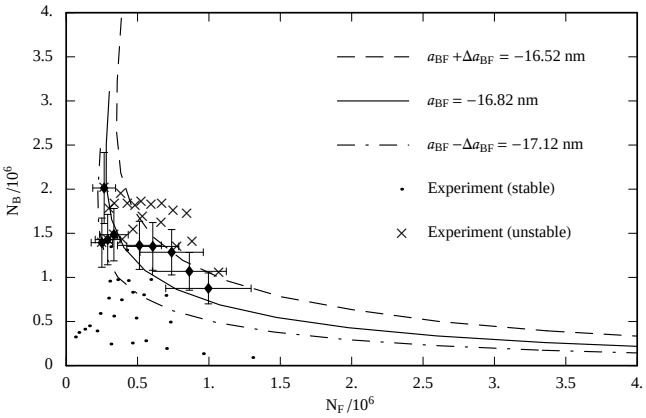


Fig. 2. Stability border for a new value $a_{BF} = (-16.82 \pm 0.30)$ nm of the ^{87}Rb–^{40}K mixture in the Hamburg experiment.[6] All three lines are obtained by numerically integrating the fermionic density (2). Those stable (unstable) points, which are located above (below) the solid line, are equipped with error bars indicating a relative uncertainty of 20% and 30% for N_B and N_F, respectively, according to Figure 3 of Ref.[6]

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