Floquet–Feshbach Resonance

Sejung Yong

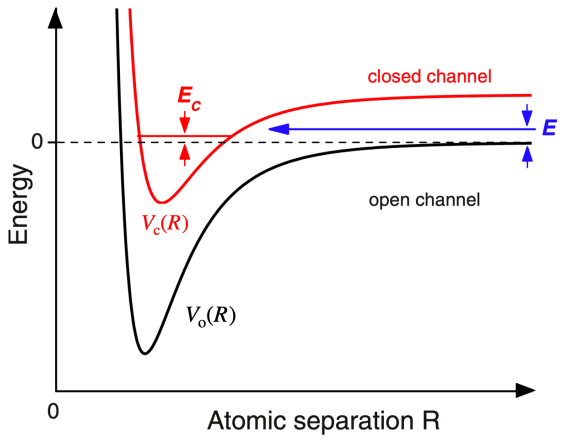
AK Theory of Ultracold Quantum Gases, Department of Physics, RPTU Kaiserslautern

Group Seminar of AG Theoretical Quantum Optics, Department of Physics, RPTU Kaiserslautern
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RPTU Rheinland-Pfälzische Technische Universität Kaiserslautern Landau

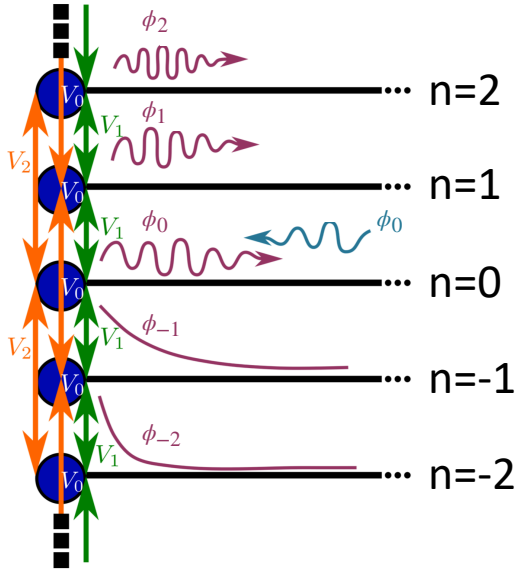
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Feshbach Resonance



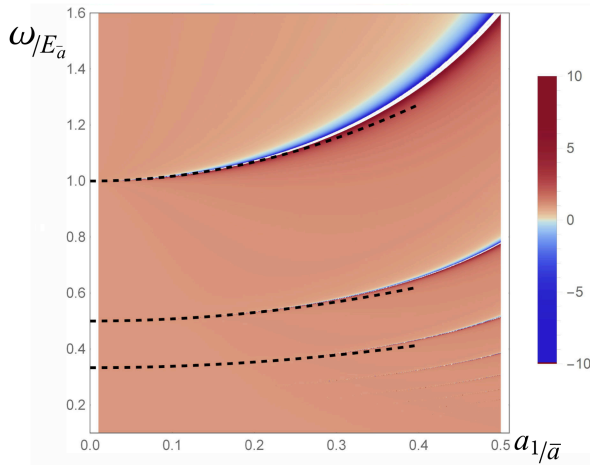
Two-channel model [1]

Floquet Scattering Theory



Visualisation [2]

Floquet Feshbach Resonance



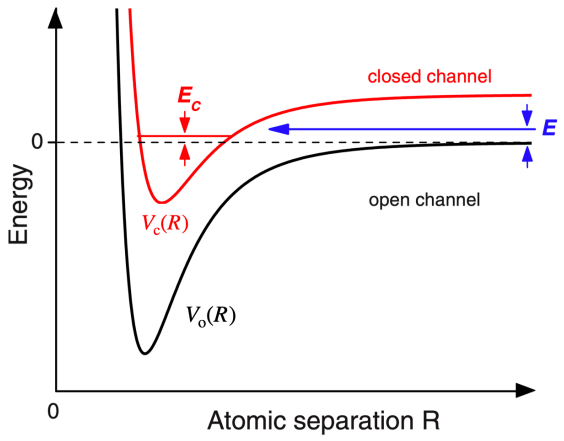
Periodic driving: scattering length [3]

RPTU

[1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010)
[2] C. Dauer, Diploma thesis, RPTU Kaiserslautern (2018)
[3] C. Dauer, A. Pelster, and S. Eggert, Phys. Rev. Lett. **135**, 033402 (2025)

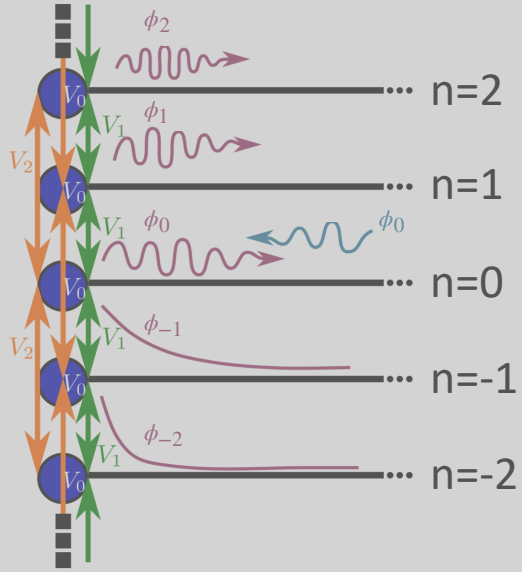
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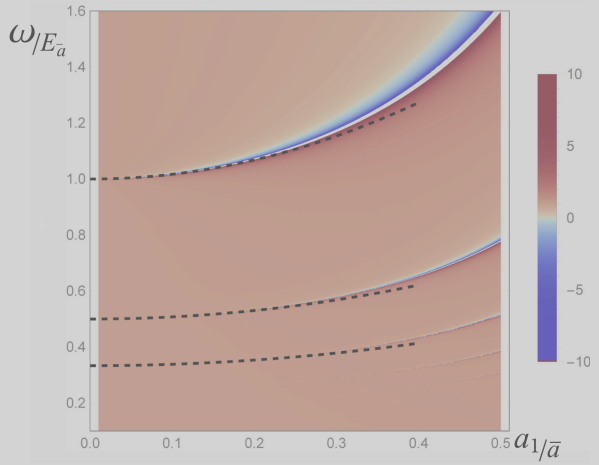
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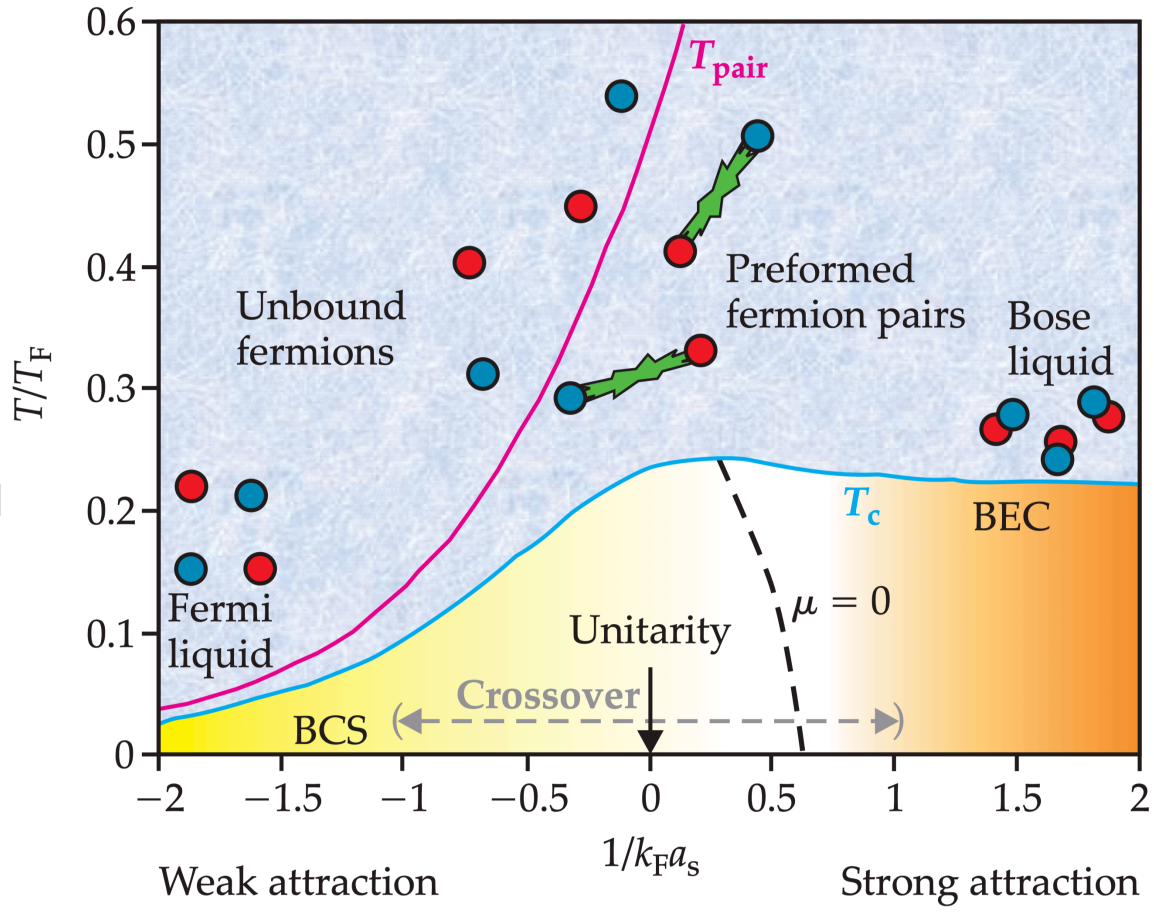
Feshbach Resonance: Motivation

- Hamiltonian:

$$\hat{H}(\lambda) = \hat{H}^{(0)} + \hat{V}(\lambda)$$

λ: control knob of interaction (e.g., B)

- Example:
  - Crossover between BCS- and BEC-superfluid phases along interaction parameter $1/k_{\mathrm{F}}a_{\mathrm{s}}$



[1] C.A.R. Sá de Melo, Physics Today **61**, 45 (2008)

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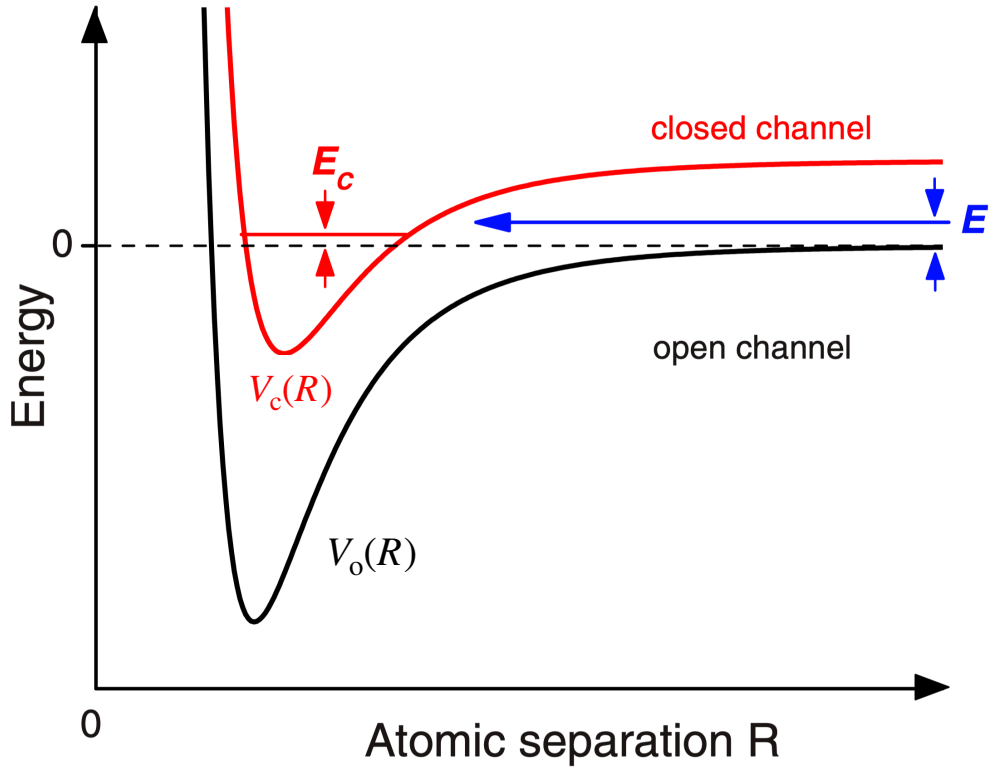
Two-Channel Feshbach Resonance

- Principle:

  coupling between open and closed channel

- Resonance condition:

  energy E in open channel resonantly couples to binding energy E_c of closed channel



[1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010)

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Elimination of Closed Channel

- Notation:

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

Feshbach projection

$$\begin{bmatrix} \hat{H}_{oo} & \hat{H}_{oc} \\ \hat{H}_{co} & \hat{H}_{cc} \end{bmatrix} \begin{bmatrix} |\psi_o\rangle \\ |\psi_c\rangle \end{bmatrix} = E \begin{bmatrix} |\psi_o\rangle \\ |\psi_c\rangle \end{bmatrix}$$

$\hat{P}_o$: open channel projection operator

$\hat{P}_c = \hat{1} - \hat{P}_o$: closed channel projection operator

$$\hat{H}_{oo} = \hat{P}_o \hat{H} \hat{P}_o$$
$$\hat{H}_{oc} = \hat{P}_o \hat{H} \hat{P}_c$$
$$\hat{H}_{co} = \hat{P}_c \hat{H} \hat{P}_o$$
$$\hat{H}_{cc} = \hat{P}_c \hat{H} \hat{P}_c$$

$$|\psi_o\rangle = \hat{P}_o|\psi\rangle$$
$$|\psi_c\rangle = \hat{P}_c|\psi\rangle$$

- Solving $|\psi_c\rangle$ for $|\psi_o\rangle$ yields effective Schrödinger equation for $|\psi_o\rangle$

$$\hat{H}_F(E)|\psi_o\rangle = E|\psi_o\rangle$$

- *Feshbach Hamiltonian*

$$\hat{H}_F(E) = \hat{H}_{oo} + \hat{H}_{oc}\left[E - \hat{H}_{cc}\right]^{-1}\hat{H}_{co}$$

} nonlinear equation for E

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Effective Interaction and Feshbach Resonance

- Effective interaction as scattering potential

coupling between channels

effective scattering potential

$$\hat{H}_{\mathrm{F}}(E) = \hat{H}_{\mathrm{oo}} + \hat{H}_{\mathrm{oc}}\left[E - \hat{H}_{\mathrm{cc}}\right]^{-1}\hat{H}_{\mathrm{co}} = \hat{H}^{(0)}_{\mathrm{oo}} + \hat{V}_{\mathrm{eff}}(E)$$

Feshbach resonance at $E - \left(E_c + \delta E_{\mathrm{c}}\right) = 0$

open channel kinetic energy

$$\hat{H}^{(0)}_{\mathrm{oo}} = \hat{H}_{\mathrm{oo}} - \hat{V}_{\mathrm{oo}}$$
$$\hat{V}_{\mathrm{eff}}(E) = \hat{V}_{\mathrm{oo}} + \hat{H}_{\mathrm{oc}}\left[E - \hat{H}_{\mathrm{cc}}\right]^{-1}\hat{H}_{\mathrm{co}}$$

$$\hat{T}(E) = \hat{V}_{\mathrm{eff}}(E) + \hat{V}_{\mathrm{eff}}(E)\left[E - \hat{H}^{(0)}_{\mathrm{oo}}\right]^{-1}\hat{T}(E)$$

T-matrix (scattering amplitude)

- Energy detuning $E - \left(E_c + \delta E_{\mathrm{c}}\right)$ controls

$$\frac{1}{T(E)} \sim \frac{1}{a_{\mathrm{s}}} - \frac{1}{2} r_{\mathrm{s}} k^2 + \ldots$$

  - resonance of *scattering length* a_{s}
  - closed-open channel coupling characterised by *effective range* r_{s}

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Broad- and Narrow-Feshbach Resonance

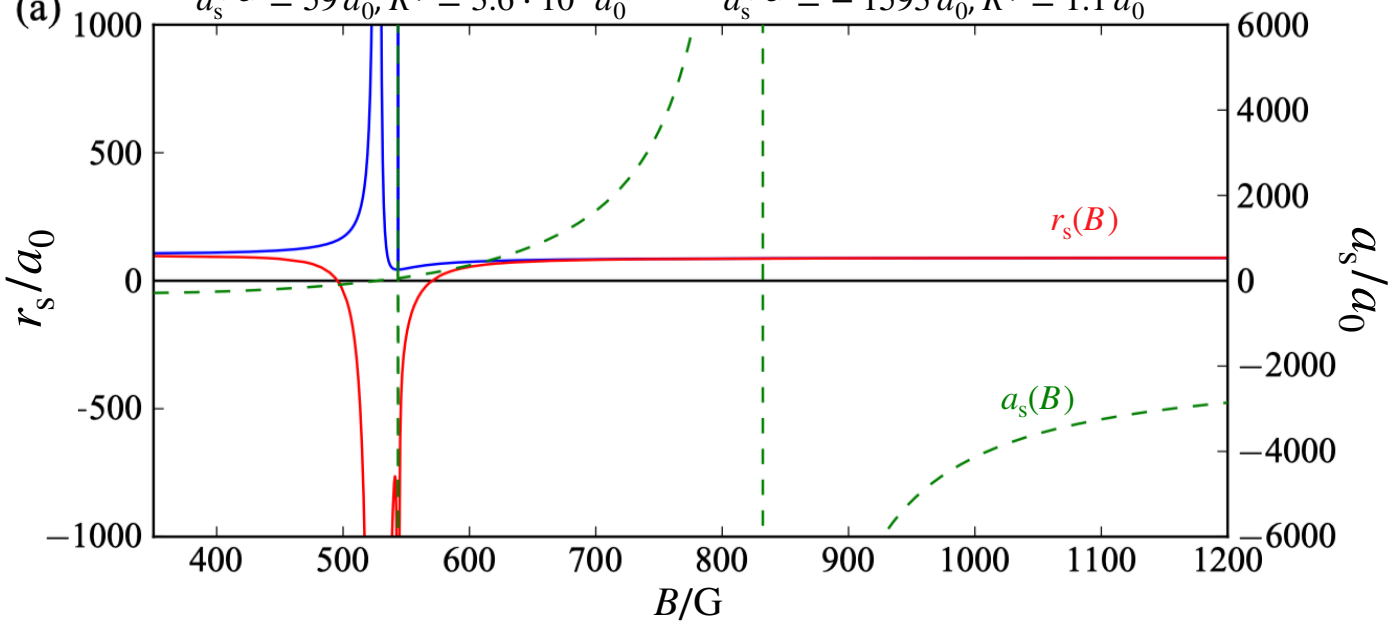
$$a_s(B) \approx a_s^{(bg)}\left(1 - \frac{\Delta}{B - B_0}\right), \quad r_s(B) \approx -2R^*\left(1 - \frac{B - B_0}{B - B_0 - \Delta}\right)^2$$

$\delta E_c := \Delta\mu(B - B_0)$: Zeeman detuning
$\Delta\mu = \mu_c - \mu_o$: magn. moment difference

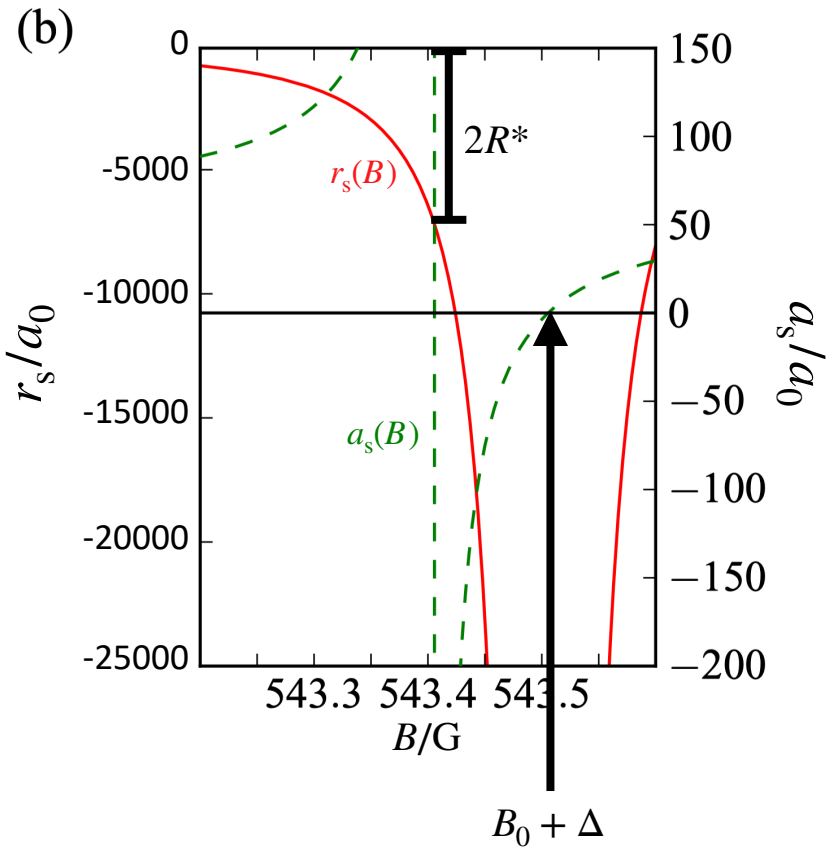
narrow FR: $|a_s^{(bg)}/R^*| \ll 1$
$B_0 = 543\,\mathrm{G}$
$a_s^{(bg)} = 59\,a_0,\ R^* = 3.6 \cdot 10^4\,a_0$

broad FR: $|a_s^{(bg)}/R^*| \gg 1$
$B_0 = 843\,\mathrm{G}$
$a_s^{(bg)} = -1593\,a_0,\ R^* = 1.1\,a_0$

a_0: Bohr radius

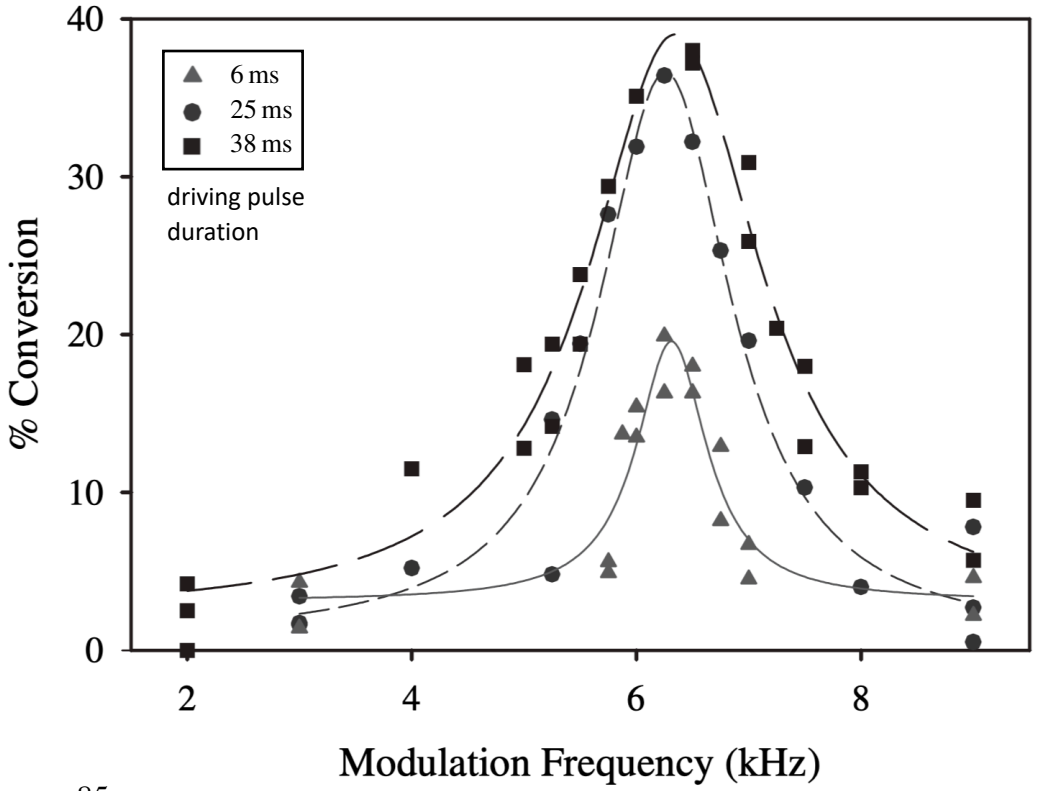


Magnetic Feshbach resonance (FR) of ^{6}Li from coupled-channel calculations [1]

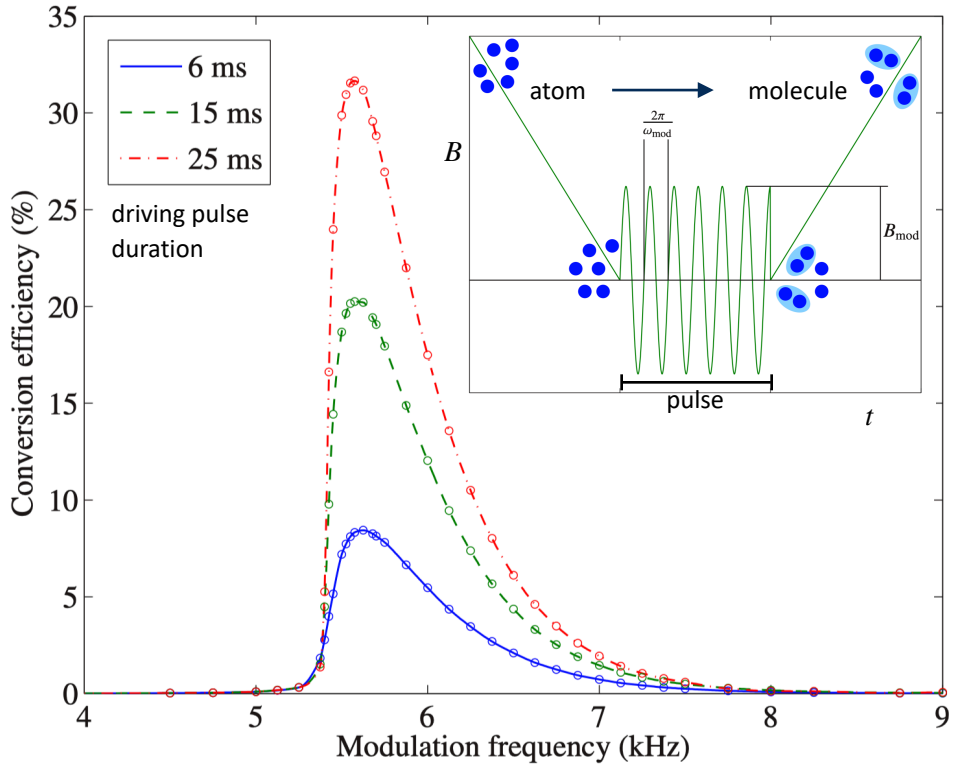


[1] C.L. Blackley, P.S. Julienne, and J.M. Hutson, Phys. Rev. Lett. **89**, 042701 (2014)

Observation of Resonances in Driven Systems



^{85}Rb-atom-molecule conversion resonance as function of driving frequency [1]



^{85}Rb-atom-molecule conversion simulation as function of driving frequency [2]

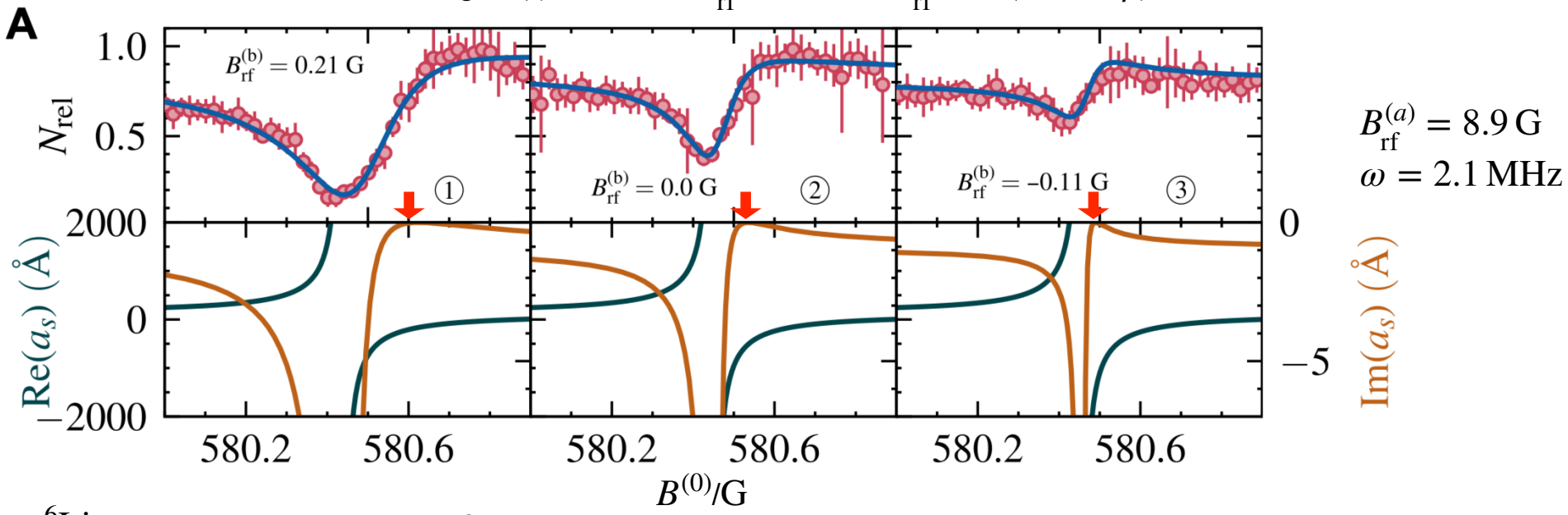
[1] S.T. Thompson, E. Hodby, and C.E. Wieman, Phys. Rev. Lett. **95**, 190404 (2005)
[2] T.M. Hanna, T. Köhler, and K. Burnett, Phys. Rev. A **75**, 013606 (2007)

Applications of Floquet–Feshbach Resonance

- Advanced experimental techniques allow precise and versatile control

  over Floquet–Feshbach resonance

two-color driving: $B(t) = B^{(0)} + B^{(a)}_{rf} \cos \omega t + B^{(b)}_{rf} \cos(2\omega t + \phi)$



above: measured ^{6}Li-atomic loss measurement from two-color driving

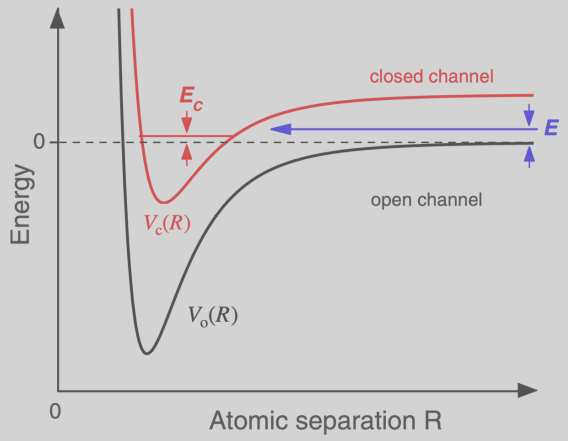
below: numerical simulation shows suppression of imaginary part(brown) of $a_s(B_0)$, while keeping real part(dark green) resonant.[1]

[1] A. Guthmann, F. Lang, L.M. Kienesberger, S.Barbosa, and A. Widera, Sci. Adv. **11**, 10 (2025)

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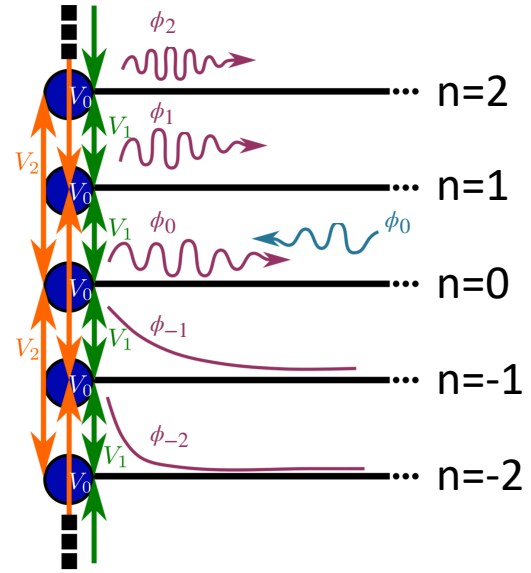
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Feshbach Resonance



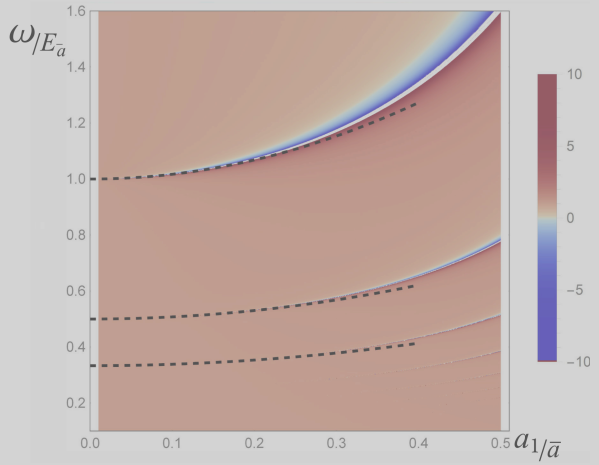
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Time-Periodic Hamiltonian

- Hamiltonian with time-periodic two-body scattering $\hat{V}(t) = \hat{V}(t+T)$

$$\hat{H}(t) = \frac{\hbar^2 \hat{\vec{k}}^2}{2M_\mathrm{r}} + \hat{V}(t)$$

$\vec{k}$: 2-body relative momentum
M_r: reduced mass

unperturbed Hamiltonian (scattering energy)

time-periodic perturbative scattering potential

- Floquet theorem

$$\hat{H}(t) = \hat{H}(t+T) \quad \Longrightarrow \quad \hat{H}(t)\,|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle \text{ has solution}$$

$$|\psi(t)\rangle = \sum_\alpha c_\alpha e^{-i\epsilon_\alpha t/\hbar}\,|u_\alpha(t)\rangle, \quad |u_\alpha(t)\rangle = |u_\alpha(t+T)\rangle$$

ϵ_α: quasi-energy
$|u_\alpha(t)\rangle$: Floquet mode

- Instead of Schrödinger equation, consider *quasi-energy eigenvalue problem*

$$\left(\hat{H}(t) - i\hbar\frac{\partial}{\partial t}\right)|u_\alpha(t)\rangle = \epsilon_\alpha\,|u_\alpha(t)\rangle$$

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Floquet Steady State

- Fourier transform of quasi-energy equation

$$\left(\hat{H}(t) - i\hbar\frac{\partial}{\partial t}\right)|u_\alpha(t)\rangle = \epsilon_\alpha |u_\alpha(t)\rangle$$

Fourier transform

$$\hat{H}(t) = \sum_{n\in\mathbb{Z}} e^{-in\omega t}\hat{H}^{(n)}$$

$$|u_\alpha(t)\rangle = \sum_{n\in\mathbb{Z}} e^{-in\omega t}|u_\alpha^{(n)}\rangle$$

$$\sum_n e^{-in\omega t}\sum_{n'}\hat{H}^{(n-n')}|u_\alpha^{(n')}\rangle - \sum_n n\hbar\omega e^{-in\omega t}|u_\alpha^{(n)}\rangle = \epsilon_\alpha\sum_n e^{-in\omega t}|u_\alpha^{(n)}\rangle$$

component

$$\sum_{n'\in\mathbb{Z}}\left(\hat{H}^{(n-n')} - n\hbar\omega\delta_{n,n'}\right)|u_\alpha^{(n')}\rangle = \epsilon_\alpha|u_\alpha^{(n)}\rangle$$

convolution — modulo $\hbar\omega$

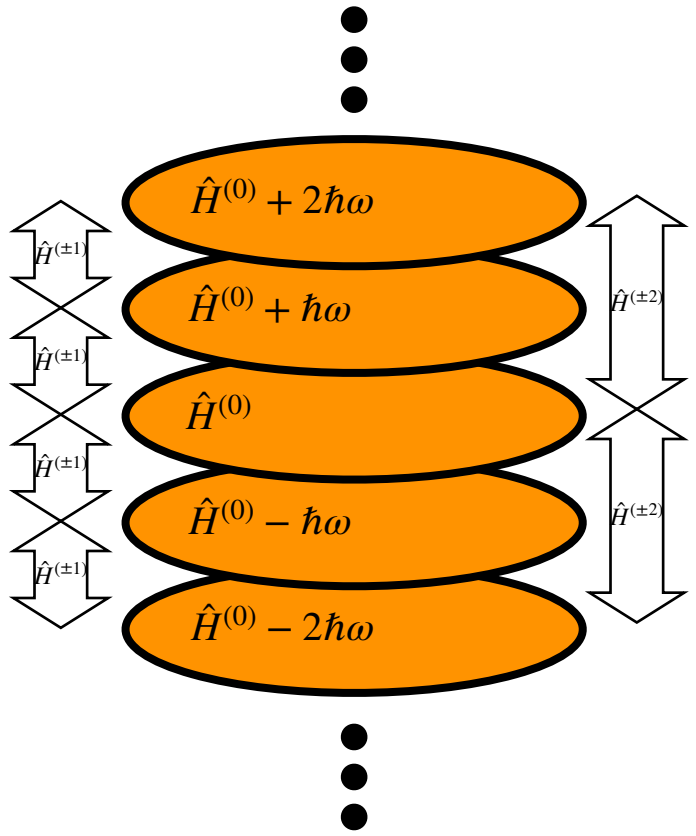
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Quasi-Energy Operator

$$\hat{\mathscr{H}}(t) = \hat{H}(t) - i\hbar\frac{\partial}{\partial t} \quad \Longleftrightarrow \quad \hat{\mathscr{H}}_{n,n'} = \hat{H}^{(n-n')} - n\hbar\omega\delta_{n,n'}$$

Fourier transform

$$\hat{\mathscr{H}} = \begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & \hat{H}^{(0)} + 2\hbar\omega & \hat{H}^{(-1)} & \hat{H}^{(-2)} & \hat{H}^{(-3)} & \hat{H}^{(-4)} & \ddots \\ \ddots & \hat{H}^{(1)} & \hat{H}^{(0)} + \hbar\omega & \hat{H}^{(-1)} & \hat{H}^{(-2)} & \hat{H}^{(-3)} & \ddots \\ \ddots & \hat{H}^{(2)} & \hat{H}^{(1)} & \hat{H}^{(0)} & \hat{H}^{(-1)} & \hat{H}^{(-2)} & \ddots \\ \ddots & \hat{H}^{(3)} & \hat{H}^{(2)} & \hat{H}^{(1)} & \hat{H}^{(0)} - \hbar\omega & \hat{H}^{(-1)} & \ddots \\ \ddots & \hat{H}^{(4)} & \hat{H}^{(3)} & \hat{H}^{(2)} & \hat{H}^{(1)} & \hat{H}^{(0)} - 2\hbar\omega & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$



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Floquet Scattering Problem

- Quasi-energy eigenvalue problem for scattering state

$$\sum_{n'\in\mathbb{Z}} \Bigg[\underbrace{\left\{ -\frac{\hbar^2 \Delta_{\vec{r}}}{2M_\mathrm{r}} - n\hbar\omega \right\}\delta_{n,n'}}_{=\mathscr{H}^{(0)}_{n,n'}(\vec{r})} + \underbrace{V^{(n-n')}(\vec{r})}_{=\int_0^T \frac{\mathrm{d}t}{T} e^{i(n-n')\omega t} V(\vec{r},t)} \Bigg] \phi_{\vec{k},n'}(\vec{r}) = E\phi_{\vec{k},n}(\vec{r})$$

- Scattering channel $n = 0$ leads to *Floquet scattering wave function*

$$\phi_{\vec{k},n}(\vec{r}) = \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}}\delta_{n,0} + \int \mathrm{d}^3 r' G^{(0)}(\vec{r},\vec{r}';E+n\hbar\omega) \sum_{n'\in\mathbb{Z}} V^{(n-n')}(\vec{r}')\phi_{\vec{k},n'}(\vec{r}')$$

asymptotic free state in scattering channel $n = 0$

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Floquet Scattering Problem with Driven Contact Potential

- Specialisation:

$$V^{(n-n')}(\vec{r}\,') \to g^{(n-n')}\delta(\vec{r}\,')$$

- The scattering wave function becomes

$$\phi_{\vec{k},n}(\vec{r}) = \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}}\delta_{n,0} + G^{(0)}(\vec{r},\vec{0};E+n\hbar\omega)\sum_{n'\in\mathbb{Z}} g^{(n-n')}\phi_{\vec{k},n'}(\vec{0})$$

$$= \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}}\delta_{n,0} + G^{(0)}(\vec{r},\vec{0};E+n\hbar\omega)\sum_{n'\in\mathbb{Z}} g^{(n-n')}\left[\frac{\delta_{n',0}}{(2\pi)^{3/2}} + \boxed{G^{(0)}(\vec{0},\vec{0};E+n'\hbar\omega)}\sum_{n''\in\mathbb{Z}} g^{(n'-n'')}\phi_{\vec{k},n''}(\vec{0})\right]$$

$$= \dots$$

$$G^{(0)}(E+n'\hbar\omega) = \lim_{\vec{r}\to\vec{r}\,'}\int_{|\vec{q}|\le\Lambda}\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot(\vec{r}-\vec{r}\,')}}{E+n'\hbar\omega+i0^+-\frac{\hbar^2}{2M_\mathrm{r}}q^2}$$

UV-cutoff renormalisation: done

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Iteration

- Notation:

$$\vec{\phi}_{\vec{k}} = \left[\ldots, \phi_{\vec{k},-1}(\vec{0}), \phi_{\vec{k},0}(\vec{0}), \phi_{\vec{k},1}(\vec{0}), \ldots\right]^{\mathsf{T}}$$

- Matrix equation:

$$\vec{\phi}_{\vec{k}} = \frac{1}{(2\pi)^{3/2}}\vec{e}_0 + \mathbf{G}^{(0)}(E)\mathbf{g}\vec{\phi}_{\vec{k}} \quad \Longleftrightarrow \quad \vec{\phi}_{\vec{k}} = \frac{1}{(2\pi)^{3/2}}\left[\mathbf{I} - \mathbf{G}^{(0)}(E)\mathbf{g}\right]^{-1}\vec{e}_0$$

$$\mathbf{G}^{(0)}(E) = \begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & G^{(0)}(E-\hbar\omega) & 0 & 0 & \ddots \\ \ddots & 0 & G^{(0)}(E) & 0 & \ddots \\ \ddots & 0 & 0 & G^{(0)}(E+\hbar\omega) & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

$$\mathbf{g} = \begin{bmatrix} \ddots & \ddots & \ddots & \ddots & \ddots \\ \ddots & g^{(0)} & g^{(-1)} & g^{(-2)} & \ddots \\ \ddots & g^{(1)} & g^{(0)} & g^{(-1)} & \ddots \\ \ddots & g^{(2)} & g^{(1)} & g^{(0)} & \ddots \\ \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

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Floquet T-Matrix

- The *Floquet T-matrix* for a driven contact potential

$$\mathbf{T}(E) = \mathbf{g}\left[\mathbf{I} - \mathbf{G}^{(0)}(E)\mathbf{g}\right]^{-1}$$

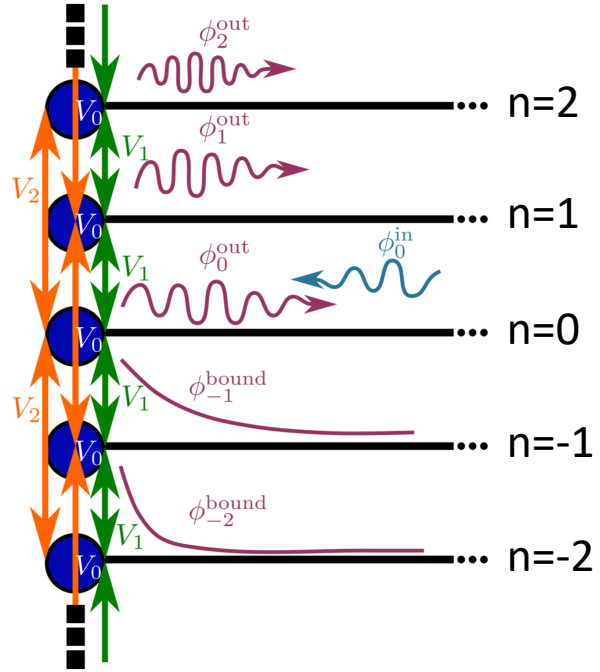
- Floquet scattering wave function

$$\phi_{\vec{k},n}(\vec{r}) = \frac{1}{(2\pi)^{3/2}}\left[e^{i\vec{k}\cdot\vec{r}}\delta_{n,0} + G^{(0)}(\vec{r},\vec{0};E+n\hbar\omega)\mathbf{T}_{n0}(E)\right]$$

- Result:

$$\phi_{\vec{k},n}(\vec{r}) \sim \begin{cases} e^{i\vec{k}\cdot\vec{r}}\delta_{n,0} + f_n(E)\dfrac{e^{ik_n(E)r}}{r}, & E+n\hbar\omega \geq 0 \\ f_n(E)\dfrac{e^{-|k_n(E)|r}}{r}, & E+n\hbar\omega < 0 \end{cases}$$

$$k_n = \sqrt{\frac{2M_\mathrm{r}}{\hbar^2}\left(E+n\hbar\omega\right)}$$



Floquet wave functions in channels $n \in \mathbb{Z}$ coupled via potential Fourier components V_ν [1]

[1] C. Dauer, Diploma thesis, RPTU Kaiserslautern (2018)

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Floquet Scattering Amplitude

- Scattering channel $n = 0$:

$$\mathbf{T}_{00} = \frac{1}{\frac{1}{g^{(0)} + \sum_{\nu \neq 0} g^{(\nu)} \Pi^{(-\nu)}} - G_0^{(0)}}$$

$$G_n^{(0)}(E) := G^{(0)}(E + n\hbar\omega)$$

- Resummed Floquet propagator

$$\Pi^{(\nu)} = \frac{L_\nu^{(\nu)}}{1 - \sum_{\mu \neq 0,\nu} L_\nu^{(\mu)} L_{\nu-\mu}^{(-\mu)} \gamma_{\nu-\mu}^{(-\mu)}}$$

ν. vertex-propagator pair at channel n

$$L_n^{(\nu)} := \frac{G_n^{(0)} g^{(\nu)}}{1 - G_n^{(0)} g^{(0)}}$$

recursion term at vertex ν and channel n

$$\gamma_n^{(\nu)} = \frac{1}{L_n^{(\nu)}} \frac{G_n^{(0)} T_{n,0}}{G_{n-\nu}^{(0)} T_{n-\nu,0}}$$

- Low-energy expansion of scattering amplitude (in limit $\Lambda \to \infty$)

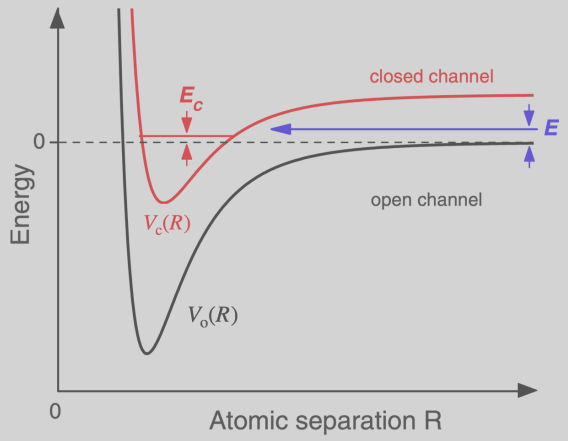
$$\frac{1}{\mathbf{T}_{00}} = \frac{1}{\left\{ g^{(0)} + \sum_{\nu \neq 0} g^{(\nu)} \Pi^{(-\nu)} \right\}_{E=0}} - \frac{\sum_{\nu \neq 0} g^{(\nu)} \frac{\partial \Pi^{(-\nu)}}{\partial k^2} \Big|_{E=0}}{\left\{ g^{(0)} + \sum_{\nu \neq 0} g^{(\nu)} \Pi^{(-\nu)} \right\}_{E=0}^2} k^2 + \mathcal{O}(k^4)$$

scattering length $a_s(\omega)$ — effective range $r_s(\omega)$

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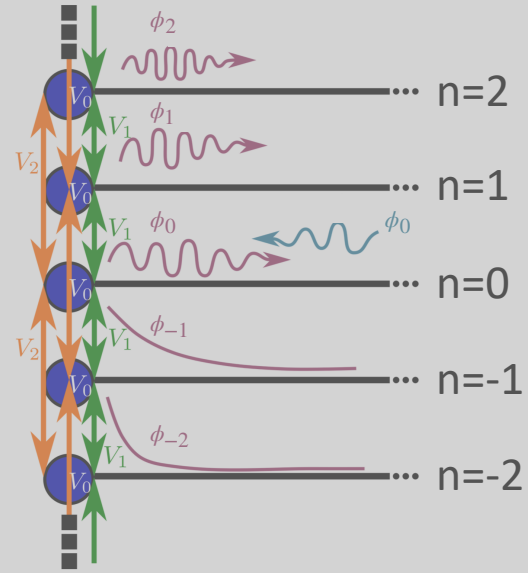
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Feshbach Resonance



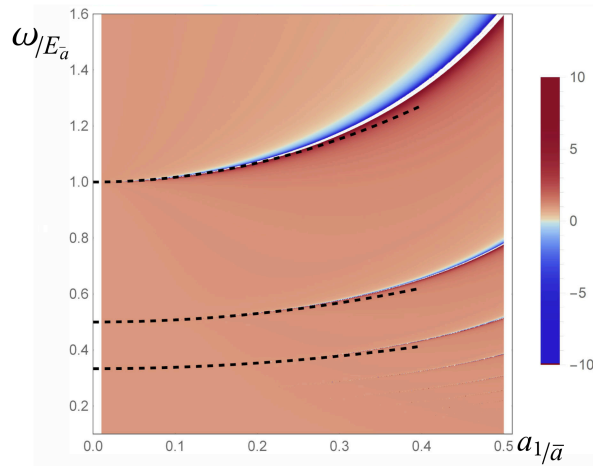
Two-channel model [1]

Floquet Scattering Theory



Visualisation [2]

Floquet Feshbach Resonance



Periodic driving: scattering length [3]

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[1] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010)
[2] C. Dauer, Diploma thesis, RPTU Kaiserslautern (2018)
[3] C. Dauer, A. Pelster, and S. Eggert, Phys. Rev. Lett. **135**, 033402 (2025)

Floquet Scattering Length

- Monochromatically driven contact potential

$$a_s(t) = a_s^{(0)} + a_s^{(1)} \cos \omega t \qquad \Longrightarrow \qquad g^{(0)} = \frac{2\pi\hbar^2}{M_r} a_s^{(0)},\ g^{(\pm 1)} = \frac{\pi\hbar^2}{M_r} a_s^{(1)},\ g^{(\nu)} = 0 \text{ elsewhere}$$

- Low-energy 0.term in $1/\mathbf{T}_{00}$: *Floquet scattering Length*

$$a_s(\omega) = a_s^{(0)} + \frac{1}{2} a_s^{(1)} \lim_{E \to 0^+} \left[\Pi^{(-1)}(E, \omega) + \Pi^{(+1)}(E, \omega) \right]$$

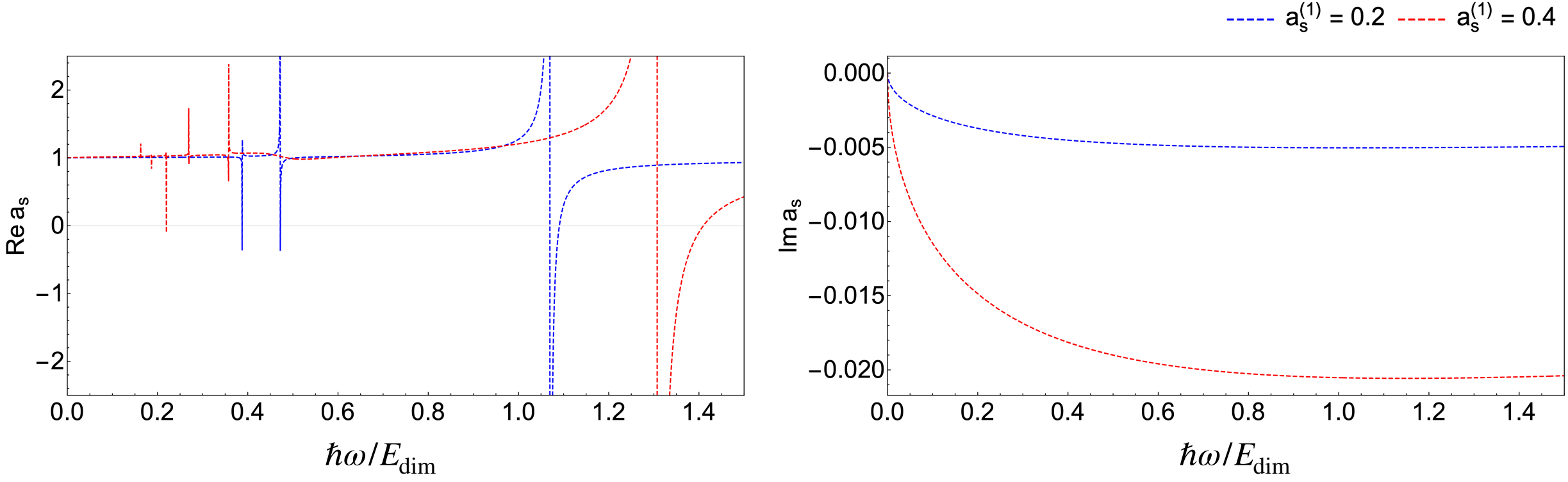
- Floquet coupling leads to continued fraction:

$$\Pi^{(\pm 1)} = \frac{L_{\pm 1}^{(1)}}{1 - L_{\pm 1}^{(1)} L_{\pm 2}^{(1)} \gamma_{\pm 2}^{(\pm 1)}} = \cfrac{L_{\pm 1}^{(1)}}{1 - \cfrac{L_{\pm 1}^{(1)} L_{\pm 2}^{(1)}}{1 - \cfrac{L_{\pm 2}^{(1)} L_{\pm 3}^{(1)}}{1 - \ddots}}}$$

$$L_n^{(\pm 1)} = \frac{G_n^{(0)} g^{(\pm 1)}}{1 - G_n^{(0)} g^{(0)}}$$

$$\gamma_n^{(\pm 1)} = \frac{1}{1 - L_n^{(1)} L_{n \pm 1}^{(1)} \gamma_{n \pm 1}^{(\pm 1)}} \text{ for } |n| > 1$$

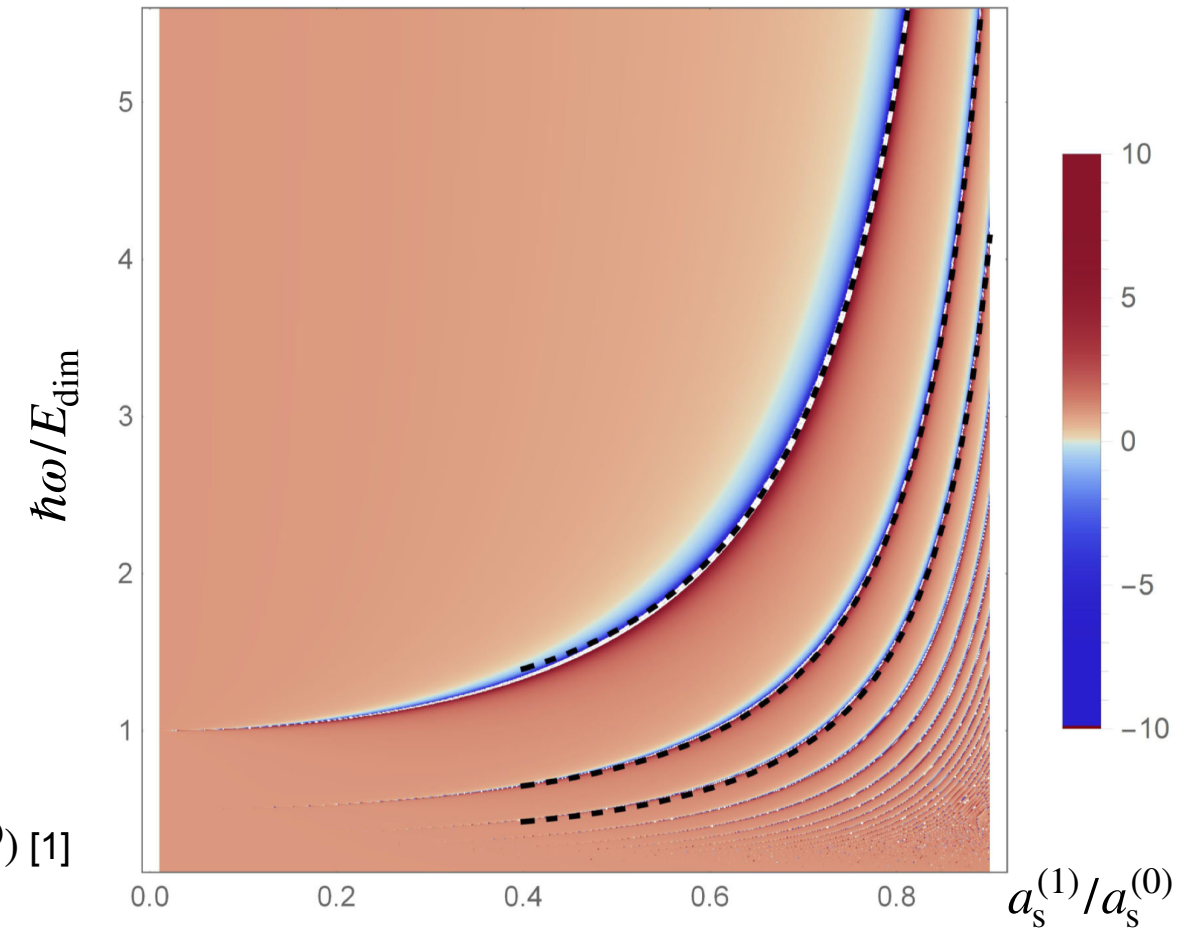
Numerical result 1



$$E_{\mathrm{dim}} = \frac{\hbar^2}{2M_{\mathrm{r}}\left[a_{\mathrm{s}}^{(0)}\right]^2}$$: dimer energy

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Numerical result 2



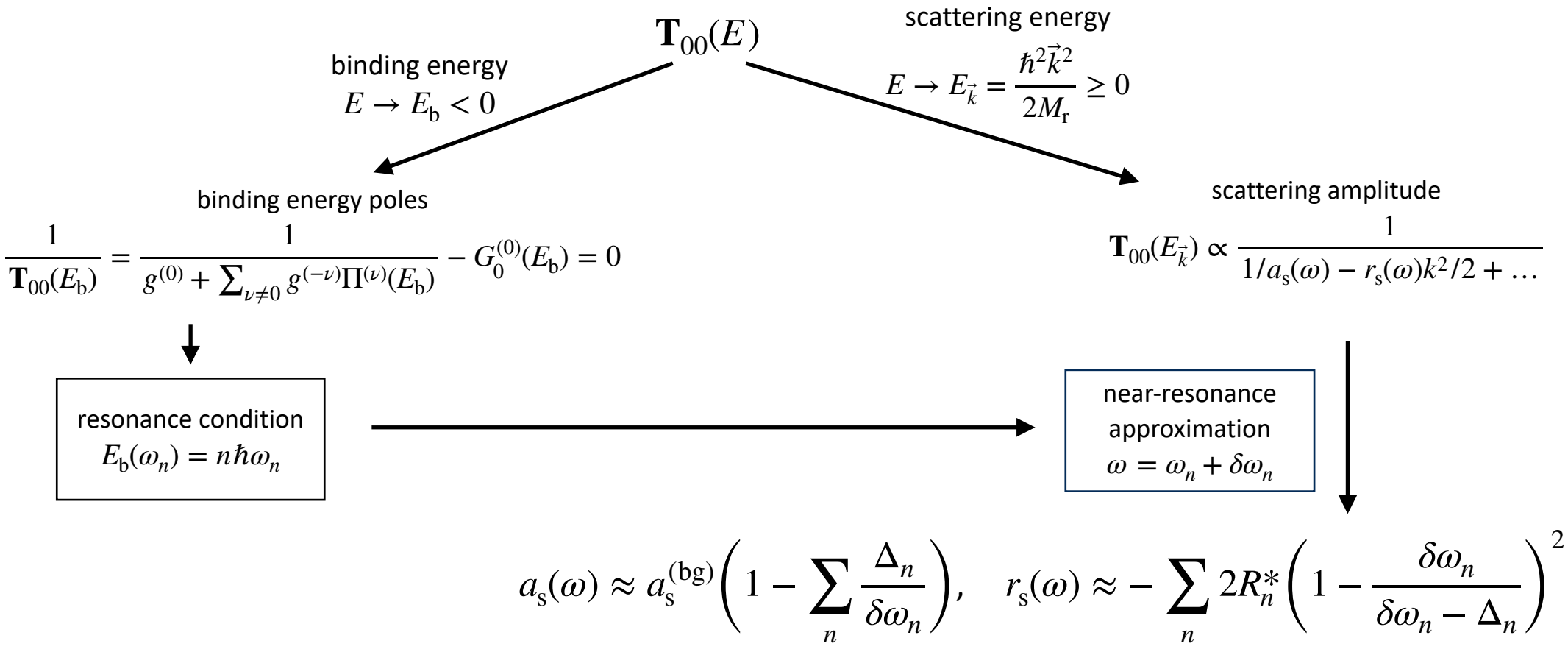
Floquet–Feshbach resonance of $\mathrm{Re}\, a_s(\omega, a_s^{(1)})$ [1]

$$E_{\mathrm{dim}} = \frac{\hbar^2}{2M_r\left[a_s^{(0)}\right]^2}\text{: dimer energy}$$

[1] C. Dauer, A. Pelster, and S. Eggert, Phys. Rev. Lett. **135**, 033402 (2025)

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Floquet–Feshbach Resonance



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Floquet Binding Energy

- T-matrix pole for $E = E_\mathrm{b} < 0$ with truncation $n = 0, -1$:

$$\frac{1}{\mathbf{T}_{00}(E_\mathrm{b})} = \frac{1}{g^{(0)} + g^{(1)} \dfrac{G_{-1}^{(0)}(E_\mathrm{b})}{1 - G_{-1}^{(0)}(E_\mathrm{b}) g^{(0)}} g^{(1)}} - G_0^{(0)}(E_\mathrm{b}) = 0$$

$$G_n^{(0)}(E) = \frac{M_\mathrm{r}}{2\pi\hbar^2} \sqrt{-\frac{2M_\mathrm{r}}{\hbar^2}(E + n\hbar\omega)}$$

- Floquet binding energy in $n = -1$

$$E_{-1} = -E_\mathrm{dim} \left(1 + \frac{1+\sqrt{2}}{\sqrt{2}} \left(\frac{a_\mathrm{s}^{(1)}}{a_\mathrm{s}^{(0)}} \right)^2 \right) + \mathcal{O}\left(\frac{a_\mathrm{s}^{(1)}}{a_\mathrm{s}^{(0)}} \right)^4$$

$$E_\mathrm{dim} = \frac{\hbar^2}{2M_\mathrm{r}\left[a_\mathrm{s}^{(0)}\right]^2}\text{: dimer energy}$$

$$\Longrightarrow \quad \omega_{-1} \approx \frac{E_\mathrm{dim}}{\hbar} \left[1 + \frac{1+\sqrt{2}}{\sqrt{2}} \left(\frac{a_\mathrm{s}^{(1)}}{a_\mathrm{s}^{(0)}} \right)^2 \right]$$

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Floquet–Feshbach Resonance Parameterisation

- General structure:

$$a_s(\omega) \approx a_s^{(\mathrm{bg})}\left(1 - \sum_n \frac{\Delta_n}{\omega - \omega_n}\right), \quad r_s(\omega) \approx -\sum_n 2R_n^* \left(1 - \frac{\omega - \omega_n}{\omega - \omega_n - \Delta_n}\right)^2$$

- Result for $n = -1$:

$$a_s^{(\mathrm{bg})} = a_s^{(0)}\sqrt{1 - \left(\frac{a_s^{(1)}}{a_s^{(0)}}\right)^2} \qquad R_{-1}^* \approx 2\frac{\left[a_s^{(0)}\right]^3}{\left[a_s^{(\mathrm{bg})}\right]^2}\left(\frac{a_s^{(0)}}{a_s^{(1)}}\right)^2$$

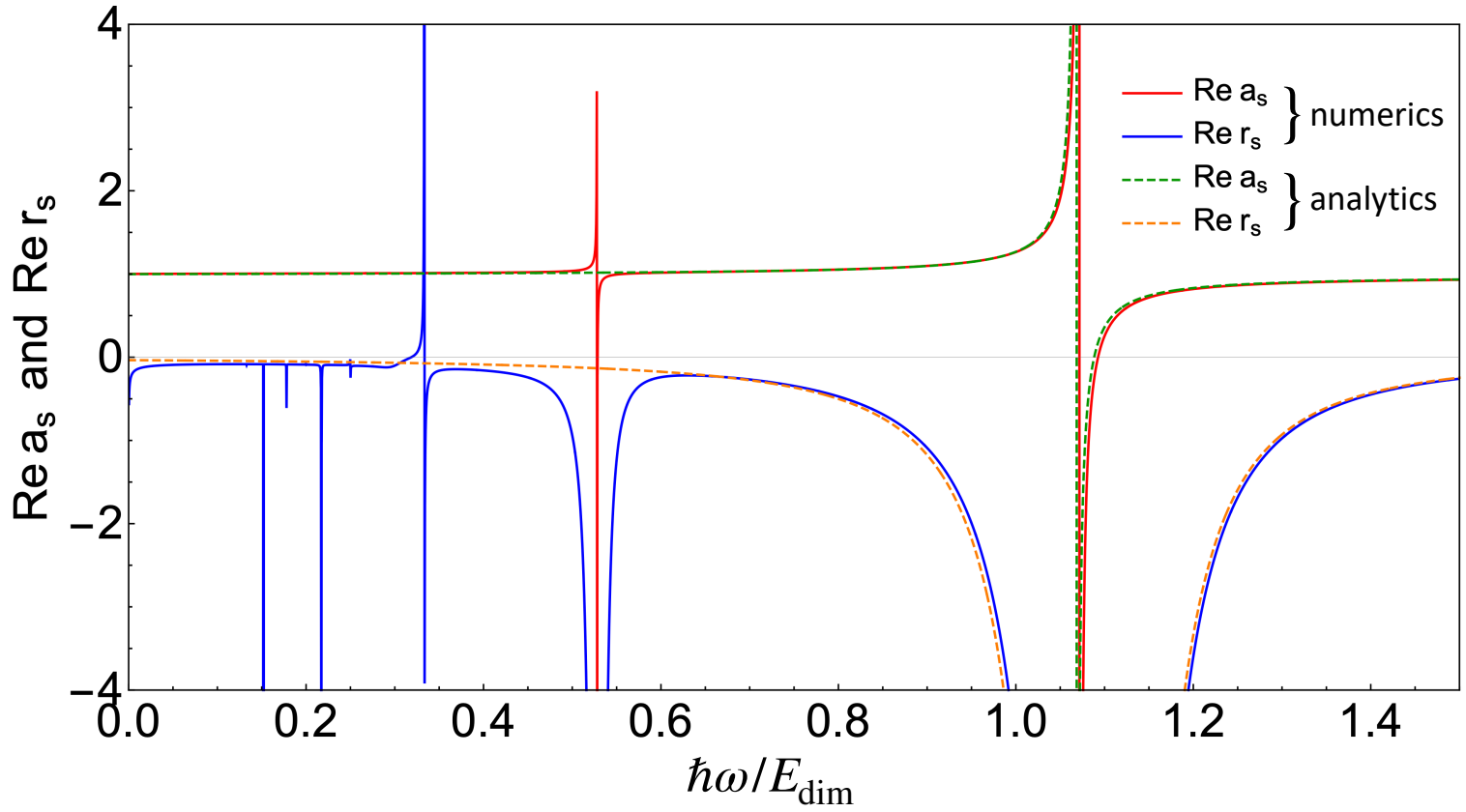
$$\Delta_{-1} \approx \frac{1}{2}\frac{E_{\mathrm{dim}}}{\hbar}\left(\frac{a_s^{(1)}}{a_s^{(0)}}\right)^2$$

$$\omega_{-1} \approx \frac{E_{\mathrm{dim}}}{\hbar}\left[1 + \frac{1+\sqrt{2}}{\sqrt{2}}\left(\frac{a_s^{(1)}}{a_s^{(0)}}\right)^2\right]$$

- Validity range: $\left|\frac{R_{-1}^*}{a_s^{(\mathrm{bg})}}\right| \sim \left(\frac{a_s^{(0)}}{a_s^{(1)}}\right)^2 \xrightarrow{|a_s^{(1)}| \ll |a_s^{(0)}|}$ narrow Feshbach resonance!

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Floquet Scattering Length and Effective Range



$$\frac{a_s^{(1)}}{a_s^{(0)}} = 0.2$$

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Outlook

- Floquet engineering
  - Multi-color driving: preventing particle loss
  - Control of resonance width: broad Floquet–Feshbach resonance?
- Periodically driven BCS-BEC crossover
  - Breaking of universality of Fermi gas [1]:
    $\left(T/T_{\mathrm{F}}, k_{\mathrm{F}}a_{\mathrm{s}}\right) \quad \rightarrow \quad \left(T/T_{\mathrm{F}}, k_{\mathrm{F}}a_{\mathrm{s}}, k_{\mathrm{F}}r_{\mathrm{s}}\right)$
  - Control of effective range: $r_{\mathrm{s}}\left(\{a_{\mathrm{s}}^{(\nu)}\}, \omega\right)$
  - Quantitative and qualitative changes in phase diagram?

[1] N. Kaschewski, A. Pelster, and C.A.R. Sá de Melo, Phys. Rev. Res. **7**, 033186 (2025)