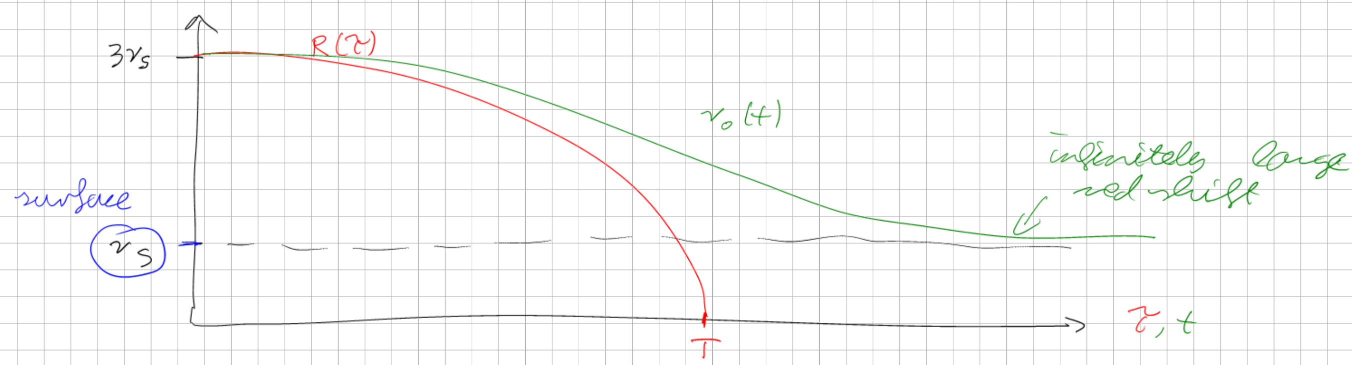
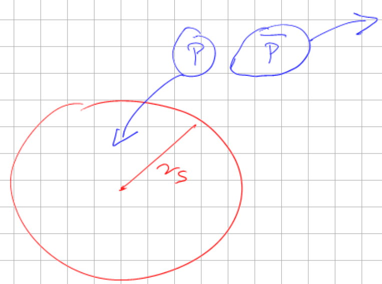
Gravitational collapse towards a black hole:



2.6.3 Properties of black holes:

- Observability: external observer sees less and less photons, but still gravitational field is observable. Surface $r = r_s$ defines a horizon, what happens inside the horizon is not observable

- Hawking effect: Isolated black hole in vacuum can radiate due to quantum fluctuations. Particle – antiparticle production near event horizon leads to particle being swallowed by black hole, antiparticle can leave



→ radiation follows a black – body follow with $T \sim \frac{M_\odot}{M} 10^{-6}$ K. Loss of matter due to radiation of particles leads to temperature increase
life time limit due to Hawking radiation $\sim \frac{M_\odot}{M} 10^{71}$ s
As $T \to \infty$, end of black hole would be an explosion.
Observability limited to life time of Universe, this should have possible with $M \approx 10^{12}$ kg $\hat{=}\ r_s = 10^{-15}$ m. Such a small black hole could be produced after Big Bang (primordial black hole).

- Energy production: throw mass into black hole

$$\frac{|\text{gravitational energy}|}{\text{rest energy}} \approx \frac{\frac{GmM}{r_s}}{mc^2} = \frac{GM}{c^2} \cdot \frac{c^2}{2GM} = 0.5$$

↑ Newton gravity

→ released due to kinetic energy, fraction of this could go into radiation energy → conversion rate ≈ 10% of rest energy
comparison with fusion: conversion rate 0.5%

matter is swallowed quite fast $\sim r_S/c$.

- Space craft falling in a black hole:
  > center of mass does not feel any gravity, as you are freely falling
  > spatial extension of astronaut: still gravitational forces are acting on astronaut
- acceleration: length distance $\Delta l = 1\,m$

$$a(r) = \frac{F(r)}{m} = \frac{GM}{r^2}, \quad |\Delta a(r)| = |a(r+\Delta l) - a(r)| = \frac{2GM}{r^3}\Delta l = \frac{c^2 r_S}{r^3}\Delta l$$

($=$ ↑ Newtonian estimate)

$$|\Delta a(r_S)| = \frac{c^2 \Delta l}{r_S^2} = \frac{c^2 \Delta l}{r_{S,\odot}^2}\left(\frac{M_\odot}{M}\right)^2 = 10^{10}\,\frac{m}{s^2}\left(\frac{M_\odot}{M}\right)^2$$

$M = M_\odot \Rightarrow |\Delta a(r_S)| = 10^{10}\,\frac{m}{s^2} \Rightarrow$ deathly forces

$M = 10^{10} M_\odot \Rightarrow |\Delta a(r_S)| = 10^{-10}\,\frac{m}{s^2} \Rightarrow$ basically no force, but travel without return

- No hair theorem: outside of black hole the black hole is only characterized by the mass M, the matter content is irrelevant, all other information is lost
  1965 John Archibald Wheeler conjectured the no-hair theorem
  $\Rightarrow$ mass M, angular momentum L, charge Q $\hat{=}$ 3 hairs
  $\rightarrow$ Kerr-Newman metric: solution of Einstein-Maxwell equations

- Observational evidence:
  > Indirect evidence: → Sagittarius A* (Sgr A*)
  Center of Milky Way: 80 stars encircle the center on an elliptical motion, Kepler's

third law you can estimate the mass $M \approx 3 \cdot 10^6 M_\odot \hat{=} r_s = 10^7 km \hat{=} \frac{1}{10}$ AU

$\approx$ mercury trajectory $\Rightarrow$ Nobel prize was given Reinhard Genzel 2020

> direct evidence: Measurements of Event Horizon Telescope allowed to take a shadow picture of 2 black holes

2019: $M87^*$ in galaxy M87, located near the Virgo supercluster at a distance $5.5 \cdot 10^7$ ly from Milky Way

$M \approx 6.5 \cdot 10^9 M_\odot \hat{=} r_s = 10^{10}$ km $= 100$ AU $\hat{=}$ solar system

2022 shadow picture of Sg A^*

- Quasars: quasi-stellar radio sources, they were long known, only 1960's detected also with visible light, red shift $z \leq 5$ large $\rightarrow$ far away known as. But as they can observed with visible light the luminosity must be large $\Rightarrow$ a few sun masses converted into visible light $\Rightarrow$ quasar = large conglomerates of $10^6 \ldots 10^{10} M_\odot$ with a black hole in center

3 Relativistic Cosmology

Last Chapter: gravitational collapse $\rightarrow$ Robertson-Walker-metric

Use again RW metric to describe expansion of Universe, galaxies as freely moving test particles, average over large scales. Here

- discuss further properties of RW metric not discussed
- space of constant curvature: how to measure it?

- discuss how to measure distance

3.1 Introduction:

Relativistic cosmology is based on 3 assumptions:

- (A1) We assume general relativity is applicable

$$\Rightarrow \quad ds^2 = \underbrace{g_{\mu\nu}}\, dx^\mu dx^\nu$$

describes space-time of Universe

- (A2) Hermann Weyl (1885 – 1955) postulated that for a cosmic observer only smeared out motion is relevant. We neglect relative velocities in our astronomical neighborhood. Each galaxy moves in a fluid, i.e. each galaxy moves along geodesics, which are orthogonal to spacelike hypersurfaces. i.e. (ct, x^1, x^2, x^3) such that each instant t the coordinates (x^1, x^2, x^3) remain constant along geodesics

$$ds^2 = c^2 dt^2 - h_{ij}\, dx^i x^j$$

t plays role of cosmic time or world time

- (A3) Cosmological principle: It generalizes the Copernican principle that the Earth is not the center of solar system. Neither the Earth, nor our solar system, nor our galaxy occupy a specific position in Universe.

$\Rightarrow$ homogeneity, isotropy $\hat{=}$ constant curvature of space

("maximally symmetric space")

$$R_{ijkl} = K\,(h_{ik}h_{jl} - h_{il}h_{jk})$$

$$= -R_{jikl} = -R_{ijlk} = R_{klij} \quad \text{valid curvature tensor}$$

$$\Rightarrow R_{jk} = R_{ijk}{}^{i}, \quad R = 6K \quad \text{is constant}$$

This statement can be converted:

Steven Weinberg, Gravitation and Cosmology

based on concept Killing vectors ≙ conserved quantities in context of GRT