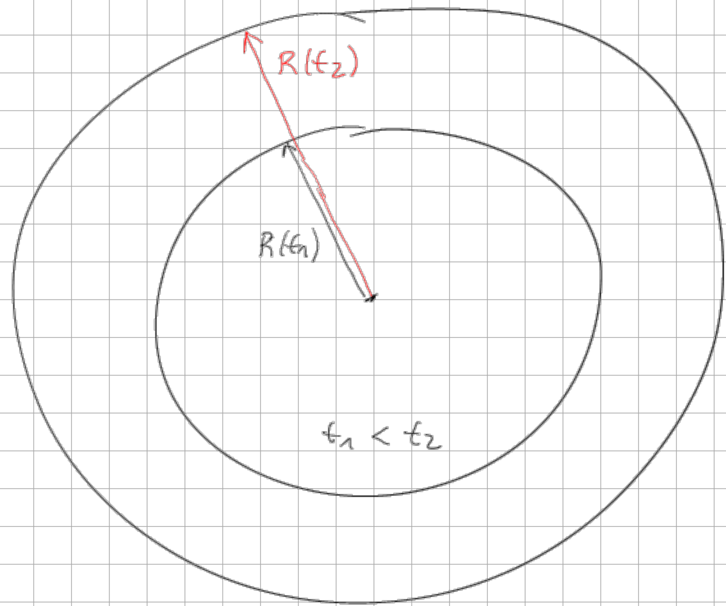
3.4 Redshift – Distance Relation:

$R(t)$ dimensionless scale factor of FLRW metric

galaxy 1 emitter — galaxy 2 receiving



$$ds^2 = c^2\,dt^2 - R^2(t)\,d\chi^2 \overset{!}{=} 0$$

photons

distance variable

$$\underbrace{r}_{\text{radial coordinate}} = \begin{cases} \sin\chi & k=+1 \\ \chi & k=0 \\ \sinh\chi & k=1 \end{cases}$$

$$\Rightarrow \quad \boxed{d\chi = \frac{c\,dt}{R(t)}}$$

The same distance between emitter and receiver:

$$\chi = \int_{t_1}^{t_2} \frac{c\,dt}{R(t)} = \int_{t_1+\delta t_1}^{t_2+\delta t_2} \frac{dt\,c}{R(t)}$$

$$\underbrace{\int_{t_1}^{t_1+\delta t_1} \frac{c\,dt}{R(t)}} + \cancel{\int_{t_1+\delta t_1}^{t_2} \frac{dt\,c}{R(t)}} = \cancel{\int_{t_1+\delta t_1}^{t_2} \frac{c\,dt}{R(t)}} + \int_{t_2}^{t_2+\delta t_2} \frac{c\,dt}{R(t)}$$

$$\Rightarrow \quad \frac{1}{R(t_1)}\cancel{c}\,\delta t_1 \approx \frac{1}{R(t_2)}\cancel{c}\,\delta t_2 \quad \xrightarrow[\nu_2 = 1/\delta t_2]{\nu_1 = \frac{1}{\delta t_1}} \quad R(t)\cdot\nu = \text{const.}$$

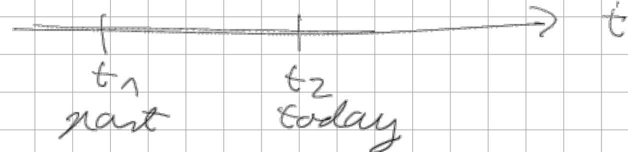
$$R(t_1)\, f_1 = \underbrace{R(t_2)}_{> R(t_1)}\, f_2 \Rightarrow f_2 < f_1$$

Cosmological red shift

$$z_c = \frac{\lambda_2}{\lambda_1} - 1 = \frac{f_1}{f_2} - 1 = \frac{R(t_2)}{R(t_1)} - 1 \xrightarrow[R(t_2) > R(t_1)]{\text{expansion}} z_c > 0$$

Quantitative analysis:

$$\underbrace{R(t_1)}_{\text{earlier time}} = R(\underbrace{t_2}_{\text{time today}} + (t_1 - t_2))$$



Taylor $R(t_2) - \dot{R}(t_2)(t_2 - t_1) + \frac{1}{2}\ddot{R}(t_2)(t_2 - t_1)^2 + \dots$

Hubble constant $H_0 = \frac{\dot{R}(t_2)}{R(t_2)}$ dimensionless deceleration parameter $q_0 = -\frac{\ddot{R}(t_2)}{\dot{R}(t_2)^2} R(t_2)$

$$R(t_1) = R(t_2)\left\{1 - H_0(t_2 - t_1) - \frac{1}{2} q_0 H_0^2 (t_2 - t_1)^2 + \dots\right\} \qquad (1)$$

$$\chi = \int_{t_1}^{t_2} \frac{c\,dt}{R(t)} = \int_{t_1}^{t_2} \frac{c\,dt}{R(t_2 + (t - t_2))} = \int_{t_1}^{t_2} \frac{c\,dt}{R(t_2) - \dot{R}(t_2)(t - t_2) + \dots}$$

$$= \int_{t_1}^{t_2} \frac{c\,dt}{R(t_2)} \left\{1 - \frac{\dot{R}(t_2)}{R(t_2)}(t - t_2) + \dots\right\}$$

$$= \frac{c}{R(t_2)} \left\{t_2 - t_1 + \frac{1}{2} \underbrace{\frac{\dot{R}(t_2)}{R(t_2)}}_{= H_0} (t_2 - t_1)^2 + \dots\right\}$$

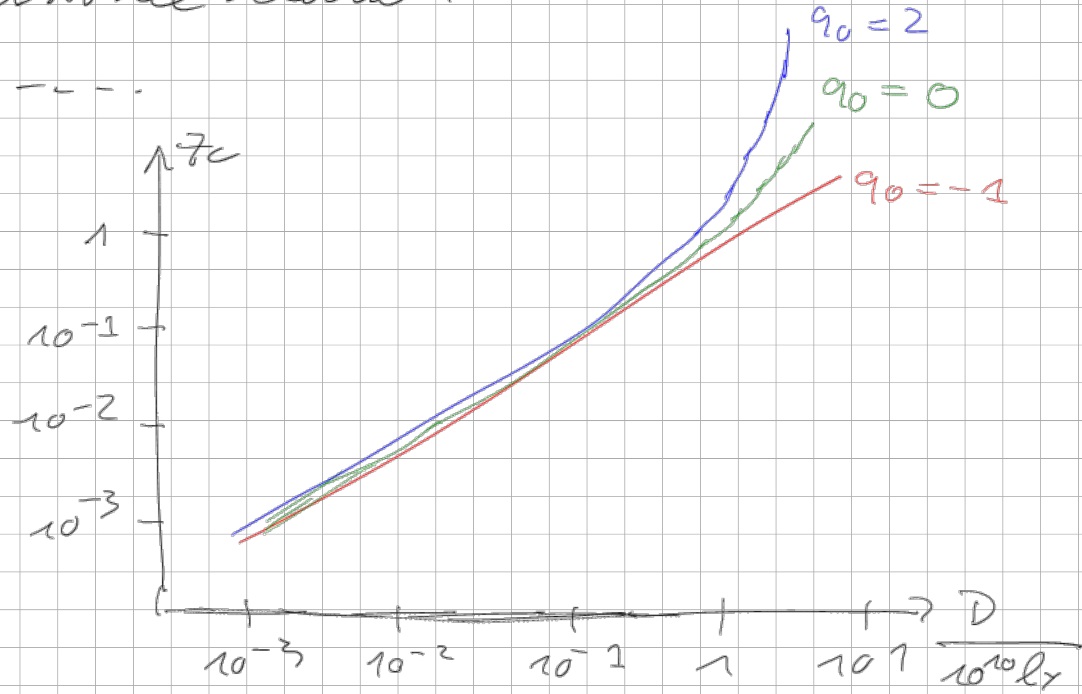
$$D = R(t_2)\chi = c(t_2 - t_1) + \frac{1}{2} c H_0 (t_2 - t_1)^2 + \dots \qquad (2)$$

⇒ Eliminate $t_2 - t_1$ via (2), express this via D

⇒ $R(t_1)$ depends on D

⇒ Cosmological redshift – distance relation

$$z_c = \underbrace{\frac{H_0}{c} D}_{\substack{\text{linear dependence} \\ \downarrow \\ H_0}} + \underbrace{\frac{1+q_0}{2} \frac{H_0^2 D^2}{c^2}}_{\substack{\text{quadratic dependence} \\ \downarrow \\ q_0,\ H_0}} + \dots$$



⇒ key quantity: distance

3.5 Cosmological Distance Ladder:

observe distant galaxy:

- cosmological redshift z $\Big\}$ q_0, H_0
- distance D

light source: absolute luminosity L $\hat{=}$ power of emitted light

on Earth: apparent luminosity ℓ $\hat{=}$ energy current density $= \frac{\text{power}}{\text{area}}$

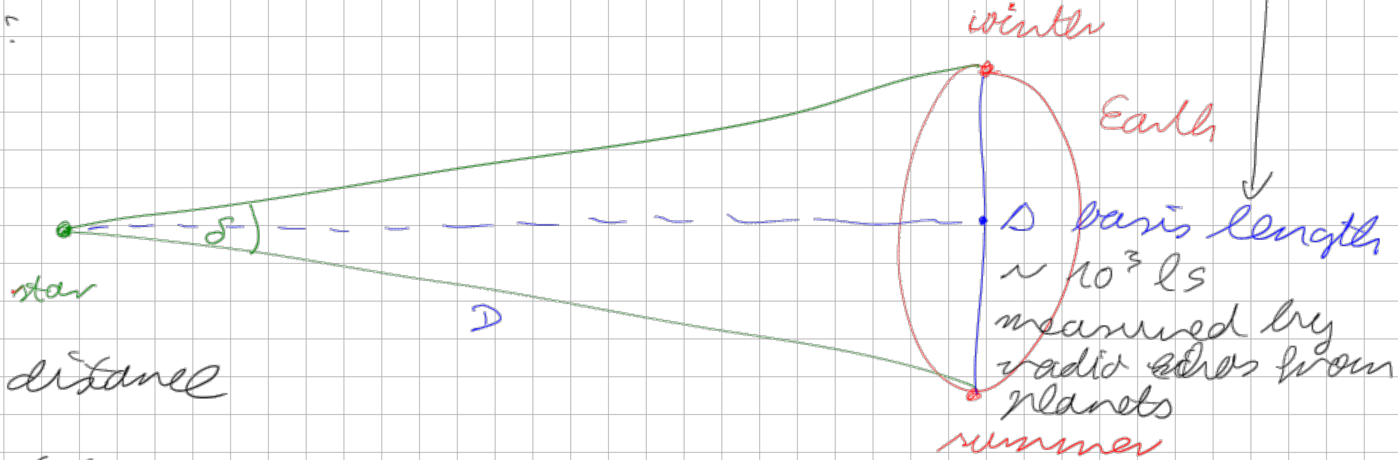
⇒ $\ell = \frac{L}{F} = \frac{L}{4\pi D_L^2}$ (*) D_L: luminosity distance

For the moment: $D_L = D$, i.e. we ignore curvature of space

3.5.1 Basic Idea and First Rung of Ladder:

In order to determine from (*) the distance D we need apparent luminosity ℓ (directly measurable) and absolute luminosity L (possibly not known).
It may happen to have light sources with particular characteristics such that their L varies only tinily, e.g. nearby stars. Then independent distance measurements allow to determine L from observed luminosity ℓ.

$\Rightarrow$ distance from triangulation:



$$\frac{\delta}{2} \simeq \tan\frac{\delta}{2} = \frac{\Delta/2}{D}$$

$$\Rightarrow D = \frac{\Delta}{\delta}$$

δ is only measurable for distance of order 30 pc, $1\,\mathrm{pc} = 3.26\,\mathrm{ly}$

far distant stars yield $\delta \simeq 0$ $\Rightarrow$ only nearby stars can be used

$\Rightarrow$ thousands of stars can be analyzed nearby Earth

$\Rightarrow$ absolute luminosities L determined

3.5.2 Second Rung of Ladder:

Hertzsprung–Russel Diagram (HRD)

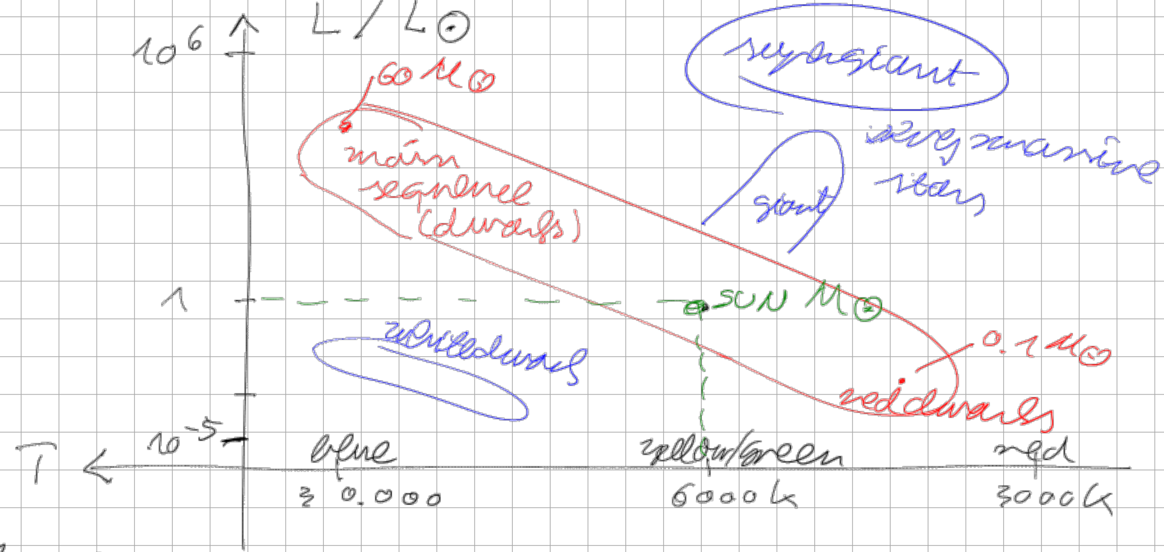
$\downarrow$ 1913

starlight has spectral distribution a la Planck, maximum corresponds to temperature

Main sequence stars: burn hydrogen to helium

Main sequence: $L = L(T)$

observable



HRD: $L=L(T)$ determined for nearby stars

Then $L=L(T)$ applied to farther distance whose distance is not accessible via triangulation (first rung)

$$\left.\begin{array}{l} \text{1) measure spectral distribution} \longrightarrow T \longrightarrow L(T) = L \\ \text{2) apparent luminosity } l \end{array}\right\} \Rightarrow D = \sqrt{\frac{L}{4\pi l}}$$

This method works works up to 100 pc

$\Rightarrow$ distances at Milky Way

3.5.3 Third Rung of Ladder:

Cepheides: radial star pulsation $\Rightarrow$ period τ $\Rightarrow$ $L = L(\tau)$

First analyse Cepheides in Milky Way: determine $L(\tau)$ from second rung

Then apply this method to Cepheids in other galaxies

$$\left.\begin{array}{l} \bullet \text{ measure } \tau \Rightarrow L(\tau) = L \\ \bullet \text{ measure } l \end{array}\right\} \Rightarrow D = \sqrt{\frac{L}{4\pi l}}$$

$\Rightarrow$ distances up to 20 MPC $\Rightarrow$ distance to local group
only accessible in 90's Hubble telescope

3.5.4 Fourth Rung of Ladder:

standard candles: supernovae Ia
white dwarfs, approaching Chandrasekhar $M_C = 1.27 M_\odot$
$\Rightarrow$ absolute luminosity L known

1) Measure distances of supernovae Ia accessible via third rung
   $\rightarrow$ discover: supernovae Ia have same L
2) far distance ~~sup~~ supernova Ia: assume have same L
   measure ℓ
   $\Big\}$ distance D

distances of 10^{10} ly are accessible

3.5.5 Fifth rung of ladder:

1) supernovae Ia of fourth rung + their redshift z
   $\Rightarrow z = z(D) \Rightarrow H_0, q_0$
2) look at more distant objects like quasars
   measure z $\xleftrightarrow{\text{deduce}}$ D