4.2 World State:

4.2.1 Defining equations:

Friedmann equation for $R(t)$:

$$\dot{R}(t)^2 - \cancel{\frac{\kappa_r}{R^2(t)}} - \frac{\kappa_m}{R(t)} - \frac{\Lambda}{3} R(t)^3 = -k \quad , \quad \dot{} \;\hat{=}\; \frac{1}{c}\frac{d}{dt}$$

neglected

Evaluate this at present time $t = t_0 \;\hat{=}\; x = 1$:

$$R_0 = R(t_0), \quad \frac{H_0}{c} = \frac{\dot{R}(t_0)}{R(t_0)}, \quad q_0 = -\frac{\ddot{R}(t_0)\, R_0}{\dot{R}^2(t_0)}$$

$$x(\tau) = \frac{R(t)}{R(t_0)} \quad , \quad \tau = H_0 t$$

$$\boxed{\left(\frac{dx}{d\tau}\right)^2 - \frac{\Omega_m}{x} - \Omega_\Lambda x^2 = -\Omega_k \quad (*)}$$

dimensionless Friedmann equation

1) matter: $\Omega_m = \dfrac{\kappa_m c^2}{H_0^2 R_0^3}$, $\kappa_m = \dfrac{8\pi G}{3c^2} R_0^3\, \rho_{mat}(t_0)$

$$\Rightarrow \quad \Omega_m = \frac{\rho_{mat}(t_0)}{\rho_{crit}(t_0)} \quad , \quad \rho_{crit}(t_0) = \frac{3H_0^2}{8\pi G} = 0.93 \cdot 10^{-26}\ \frac{kg}{m^3}$$

historical reason for calling $\rho_{crit}(t_0)$ a critical density follows soon.

2) Λ term: $\Omega_\Lambda = \dfrac{\Lambda c^2}{3 H_0^2}$

3) k term: $\Omega_k = -\dfrac{k c^2}{R_0^2 H_0^2}$

Evaluate $(*)$ at $t = t_0$: $\tau = H_0 t_0 = 1$, $x = 1$, $\dfrac{dx}{d\tau} = \dfrac{1}{R_0}\dfrac{dt}{d\tau}\dfrac{dR_0}{dt} = \dots = 1$

$$\Rightarrow \quad \Omega_m + \Omega_\Lambda + \Omega_k = 1 \qquad (1)$$

Another relation follows from differentiating Friedmann equation with respect to time, making it dimensionless and evaluating it for $t = t_0$: $$q_0 = \frac{\Omega_m}{2} - \Omega_\Lambda \qquad (2)$$

In total we have 5 parameters: $\Omega_m, \Omega_k, \Omega_\Lambda, H_0, q_0$

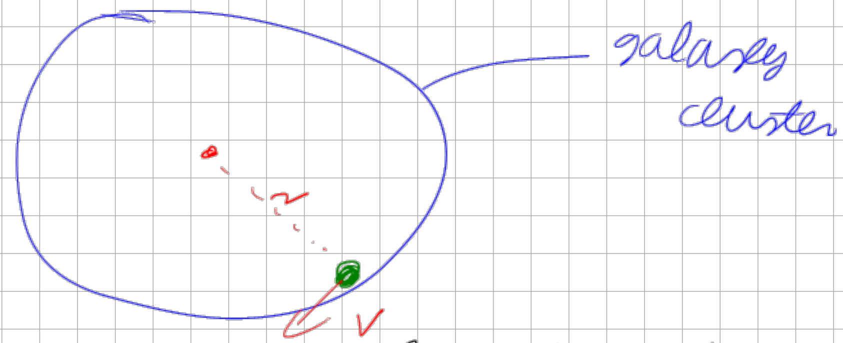
4.2.2 Parameter values:

redshift-distance relation: $H_0 = (72 \pm 8)\,\frac{km/s}{Mpc}$, $q_0 = -0.5 \pm 0.15$

$\Rightarrow$ one additional parameter needs to be fixed by observations:

One possibility: Ω_m

observe a galaxy at the border of a galaxy cluster



$$F_Z = F_G \Rightarrow \frac{\cancel{m} v^2}{\cancel{r}} = \frac{G M \cancel{m}}{r^{\cancel{2}}}$$

$$M_{\text{galaxy cluster}} = M(v, r) = \frac{v^2 r}{G}$$ mass within cluster, observable via gravity

weighted with number of galaxy clusters

$$\Rightarrow \quad \Omega_m = 0.3 \pm 0.1$$

$$\Rightarrow \quad \Omega_\Lambda = 0.7 \pm 0.1, \quad \Omega_k = 0 \pm 0.06$$

small curvature energy, Universe nearly flat

Ω_k quite small follows independently:
Boomerang and Maxima-1 experiment
measure very precisely the anisotropy of CMB $\Longrightarrow$ $\Omega_k \approx 0$

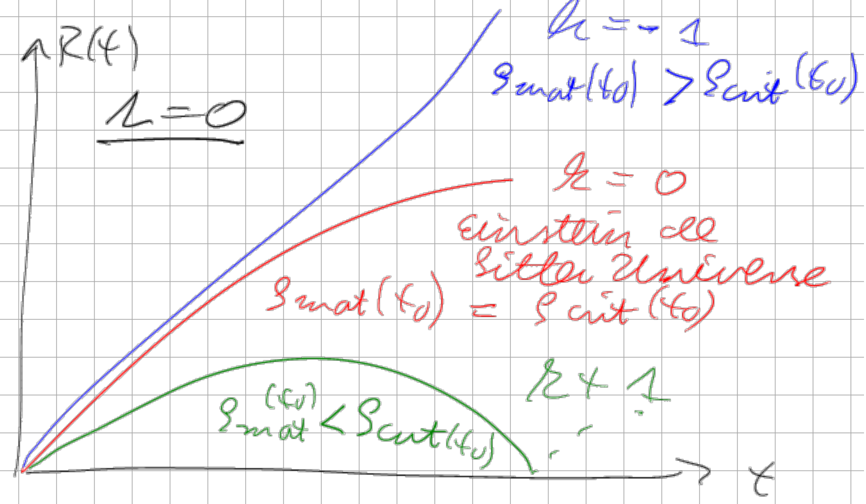
more insight needed

4.2.3 Historic remark:

Before 1997 the belief was: $\Omega_\Lambda = 0$

$$(\Omega_m, \Omega_\Lambda, \Omega_k, q_0) = (1, 0, 0, 0.5)$$

$$\Lambda = 0: \quad \Omega_k = 1 - \underbrace{\Omega_m}_{= \frac{\varrho_{mat}(t_0)}{\varrho_{crit}(t_0)}}$$



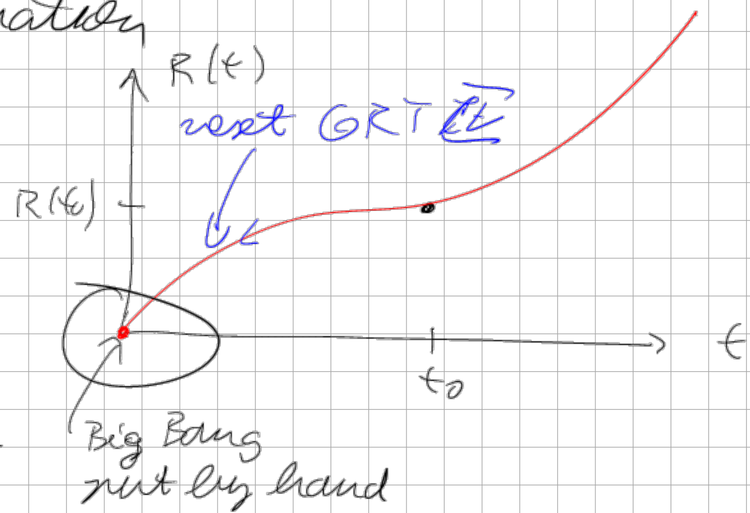
4.2.4 Temporal Evolution:

R(t) follows from solving Friedmann equation

$$\tau = H_0 t = \int_0^{x(\tau)} \frac{dx'}{\sqrt{\frac{\Omega_m}{x'} + \underbrace{\Omega_\Lambda x'^2}_{\text{largest term}} + \Omega_k}}$$

present day: $t = t_0 \qquad x(\tau = 1) = 1$

$$t_0 = \int_0^1 \frac{dx}{dx/dt} = \frac{1}{H_0} \underbrace{\int_0^1 \frac{dx}{\sqrt{\frac{\Omega_m}{x} + \Omega_\Lambda x^2 + \Omega_k}}}_{\approx 0.964} \approx 13 \cdot 10^9 \, a$$



$\Rightarrow \quad t_0 \approx \frac{1}{H_0}$ very good approximation surprisingly

Surprisingly the Milky Way is nearly as old as the Universe!

- galaxy $\Rightarrow$ stars are needed
- supernova: production of heavy elements, i.e. beyond iron
- nuclei can capture neutrons

$\longrightarrow$ abundance of certain isotopes

- $\underbrace{^{235}U}_{\text{decays}} / ^{238}U \approx 1.65$ abundance ratio at the instant of creation

$\longrightarrow$ decrease in abundance ratio : nuclear cosmochronology

- today: $0.007 \longrightarrow$ age of Milky Way: $13.6 \cdot 10^9 a$

compared to age of Universe: $13.81 \cdot 10^9 a$

$\Rightarrow$ Milky Way emerged quite soon after Big Bang

What is the radius of todays visible Universe:

FLRW metric: $d\vartheta = d\varphi = 0 \rightarrow$ light goes along straight lines

$$ds^2 = c^2 dt^2 - R^2(t)\, d\chi^2 = 0 \Rightarrow d\chi = \frac{c\,dt}{R(t)}$$

Horizon of the Universe today:

$$D_0 = R_0 \cdot \chi(t_0) = R_0 \int_0^{t_0} \frac{c\,dt}{R(t)} = \frac{c}{H_0} \int_0^1 \frac{dx}{x \frac{dx}{d\tau}}$$

$$= \frac{c}{R_0} \int_0^1 \frac{dx}{\sqrt{\Omega_m x + \Omega_r x^4 + \Omega_\Lambda x^2}} \approx 44 \cdot 10^9\, ly$$

factor 3 larger horizon than naively expected

Qualitative explanation: original distances were smaller, but light always travels with light velocity.

Solution for Olber paradoxon: observable Universe is not infinite but finite.

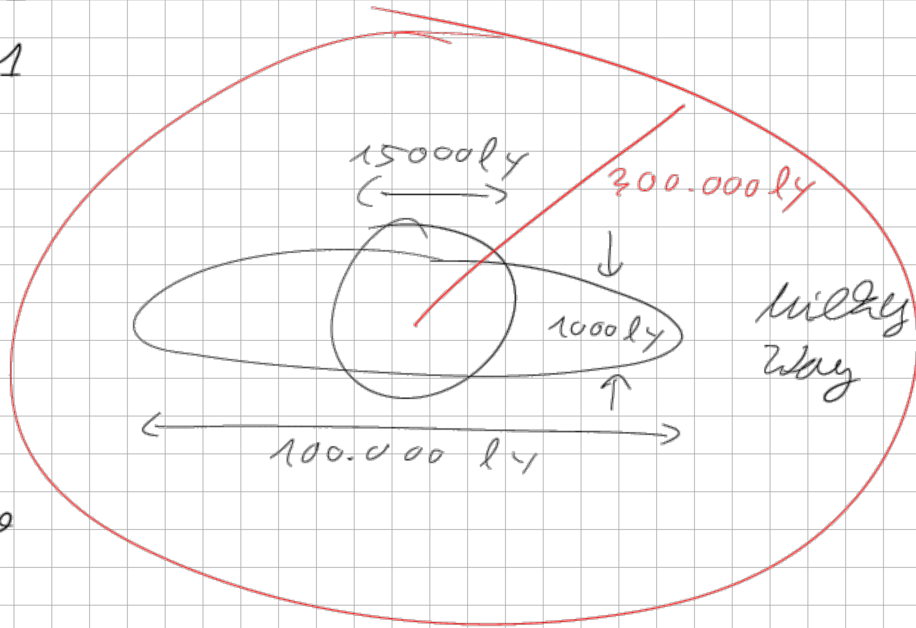
4.2.5 Mass Energy Density:

$\Omega_m = 0.3 \pm 0.1$: matter due to gravity

visible light from stars: $\Omega_m^{vis} = 0.04 \pm 0.01$

dark matter : $\Omega_m^{dark} = 0.26 \pm 0.01$

Milky Way: 95% of matter is dark



Possible candidates for dark matter:

- cold gas / cold dust cloud:
  would heat up and then become visible
- MACHOs: massive astrophysical compact halo objects => brown dwarfs
  > less than 75 M jup.: too small for hydrogen fusion
  > larger than 13 M jupit = allow for deuterium fusion
  > found between Earth and large Magellian cloud due to gravit. lensing
  → small fraction for dark matter

isotropic dark matter distribution despite anisotropic spiral galaxy

=> Non-baryonic matter:

- Hot dark matter (HDM): fast light particles
  > mass of neutrinos is too small
  > top-down evolution of Universe: first galaxy cluster, then galaxies, then stars
  → small fraction
- Cold dark matter (CDM):
  > bottom-up evolution of Universe
  > WIMPs: weakly interacting massive particles
  > supersymmetric partner particles
- Self-interacting dark matter (SIDM)
  > e.g. axion (no electric charge, no spin)
  > QCD needs CP violation, yields electric dipole moment of up to 10^{-16} e cm
  > experiment: $< 10^{-25}$ e cm
  > explainable via introducing axions, but not found so far

Note: $0.01 \leq \Omega_\nu \leq 0.05$

4.2.6 Vacuum Energy Density:

$$\Omega_\Lambda = 0.7 \Rightarrow \Lambda = \frac{3H_0^2}{c^2}\Omega_\Lambda \approx \frac{2H_0^2}{c^2}$$

$$\underbrace{\rho_\Lambda}_{\text{energy density}} = \frac{\Lambda c^2}{8\pi G} = \frac{H_0^2}{4\pi G} = 5 \cdot 10^{-26} \frac{kg}{m^3} \Rightarrow \text{experimental value}$$

theoretical explanation: vacuum fluctuations

E.g. zero-point oscillation of photon field

$$E_0 = \sum_i \frac{\hbar \omega_i}{2} \cdot \underbrace{2}_{\text{polarizations}} = \frac{V}{(2\pi)^3} \int d^3k\, \hbar\, \omega_{\vec{k}} \sim V \hbar c\, k_{max}^4$$

cut off

Note: in principle ALC particles should contribute!

What physically can $k_{max} = \frac{1}{L_{min}}$ be?

Planck length!

$$\frac{G M m}{\lambda_c} = m c^2 \Rightarrow \lambda_c = \frac{G M_P}{c^2} \overset{!}{=} \frac{\hbar}{M_P c} \Rightarrow M_P = \sqrt{\frac{\hbar c}{G}} = 1.2 \cdot 10^{19} \frac{GeV}{c^2}$$

Planck mass

$$L_P = \lambda_c = \frac{\hbar}{M_P c} = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \cdot 10^{-35}\, m$$

$$\rho_{vac} = \frac{E_0}{V c^2} = \cdots = \frac{M_P}{L_P^3} = 5 \cdot 10^{93} \frac{kg}{m^3}$$

$$\frac{\rho_\Lambda}{\rho_{vac}} = 10^{-119}$$

$\Rightarrow$ largest discrepancy between theory and experiment.

This justifies to call ρ_Λ as the dark energy.