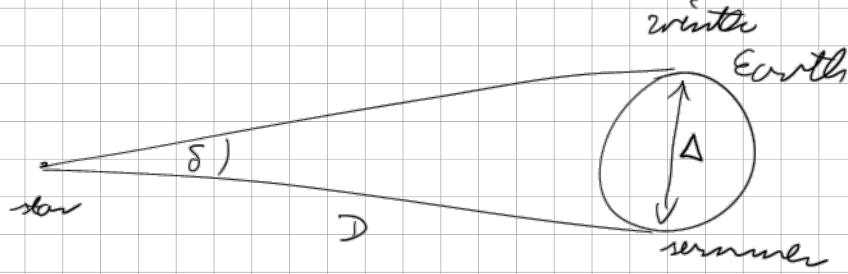
Cosmological Distance Ladder:

- first rung: triangulation

$$D = \frac{\Delta}{\delta}$$



- 2nd – 5th rung:

apparent luminosity ℓ – absolute luminosity L

3.5.6 Distance measurements in FLRW metric:

measuring D in Euclidean space $\longleftrightarrow$ measuring D_L in curved space?

$$ds^2 = c^2 dt^2 - R(t)\left\{ d\chi^2 + f(\chi)^2 \left[d\vartheta^2 + \sin^2\vartheta \, d\varphi^2 \right] \right\}$$

$$f(\chi) = \begin{cases} \sin\chi & k=1 \\ \chi & k=0 \\ \sinh\chi & k=-1 \end{cases}$$

triangularation method:

- angle $\delta \mathrel{\hat{=}} d\vartheta$
- length $\Delta \mathrel{\hat{=}} R(t) f(\chi)\, \delta$
- distance: $D = R(t)\chi$

$\Big\}$ eliminating scaling factor $R(t)$ $\Rightarrow$ $D = \frac{\Delta}{\delta} \underbrace{\frac{\chi}{f(\chi)}}$

reflects impact of spatial curvature

Observation:

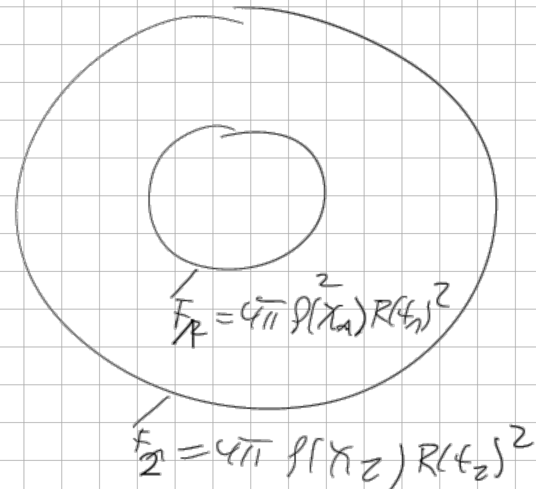
- small distances: a few pc
- spatial curvature radius $\hat{=}$ several 10^{10} ly (as determined later)

negligible curvature effects for triangulation

2nd to 5th rung:

$$\ell = \frac{L}{4\pi\, f(\chi)^2 \underset{\uparrow}{R(t_2)^2}} \underbrace{\frac{R(t_1)}{R(t_2)}}_{\text{reduced frequency}} \underbrace{\frac{R(t_1)}{R(t_2)}}_{\text{dilution of photons}} \overset{!}{=} \frac{L}{4\pi D_L^2}$$

scaling factor of today

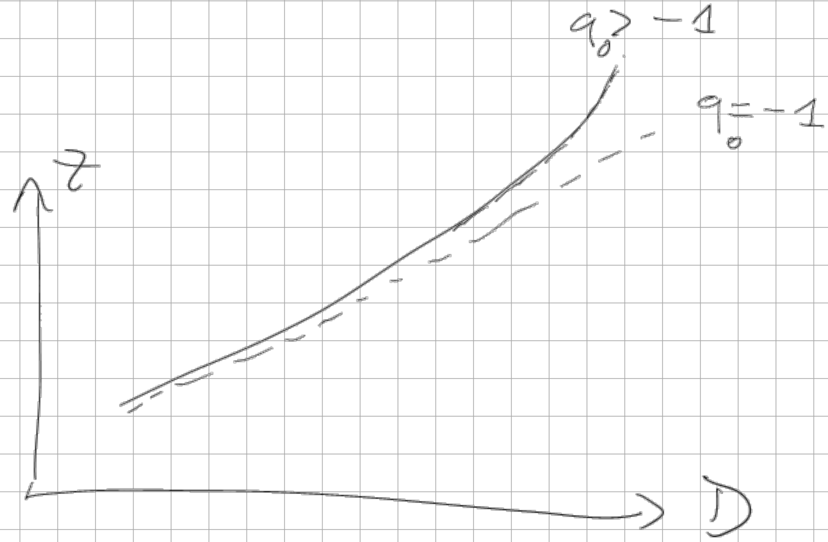


$$\Rightarrow D_L = f(\chi) \frac{R(t_2)^2}{R(t_1)}, \qquad D = \chi R(t_2), \quad z = \frac{R(t_2)}{R(t_1)} - 1$$

$$\Rightarrow D_L = D \underbrace{\frac{f(\chi)}{\chi}}_{\text{taking into account spatial curvature}} \underbrace{(1+z)}_{\text{cosmological red shift}}$$

3.5.7 Hubble and deceleration parameter:

$$H_0 = \frac{\dot{R}(t_2)}{R(t_2)} \qquad q_0 = -\frac{\ddot{R}(t_2)\, R(t_2)}{\dot{R}(t_2)^2}$$



- original Hubble parameter:

$$H_0 \approx 500 \frac{km}{s} \frac{1}{Mpc}$$

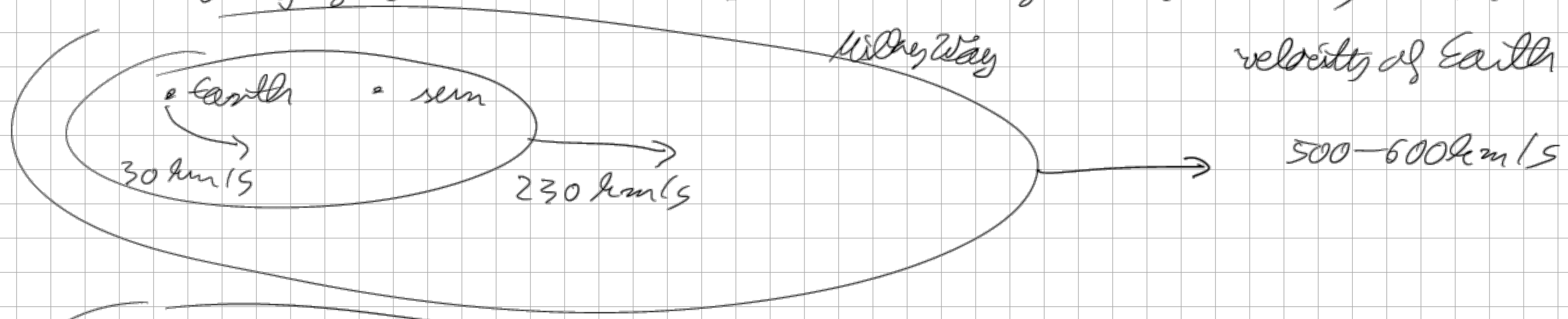
- todays value $H_0 = (72 \pm 8) \frac{km}{s} \frac{1}{Mpc}$

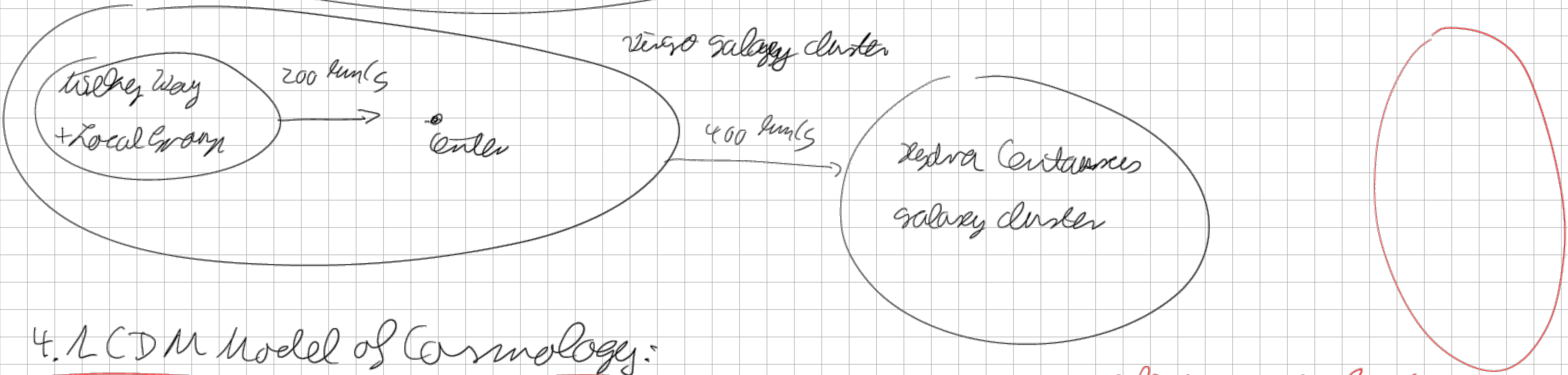
10^7 ly $\Rightarrow$ galaxies moving away from us 210 km/s

10^8 ly $\Rightarrow$ 2000 km/s

deceleration parameter: $q_0 = -0.5 \pm 0.16 \Rightarrow$ accelerated expansion

Note: velocity of Earth $\hat{=}$ 400 km/s $\hat{=}$ distance of about $3 \cdot 10^7$ ly $\Rightarrow$ at larger distances velocity of Earth negligible





4. Λ CDM Model of Cosmology:

Λ CDM model describes evolution of cosmos after Big Bang with three major components:

- a cosmological constant Λ $\hat{=}$ dark energy
- cold dark matter (CDM)
- ordinary matter

It is the simplest model that provides reasonable good account of

- Cosmic microwave background (CMB)

- large-scale structure in distribution of matter of Universe
- observed abundances of hydrogen, helium, lithium
- accelerated expansion of Universe

Model based GRT, worked out in late 90's $\Rightarrow$ concordance cosmological.

Despite successes there are still various features not understood.

4.1 World Models:

Apply Einstein's field equations as whole to the Universe. With simplifying assumptions we will solve them with FLRW metric.

4.1.1 Assumptions:

field equations with cosmological constant Λ:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

trace: $R - \frac{R}{2} 4 - \Lambda 4 = \frac{8\pi G}{c^4} T \Rightarrow R = -4\Lambda - \frac{8\pi G}{c^4} T$, $T = T_\mu{}^\mu = g^{\mu\nu} T_{\mu\nu}$

$$\Rightarrow \boxed{R_{\mu\nu} + \lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\underbrace{T_{\mu\nu}}_{\substack{\text{energy / momentum of all other} \\ \text{fields than gravity}}} - \frac{T}{2} g_{\mu\nu}\right)}$$

matter distribution: approximately homogeneous, cosmological principle

$\Rightarrow$ isotropic

$\Rightarrow$ ideal fluid: $T_{\mu\nu} = \left(\rho + \frac{P}{c^2}\right) u_\mu u_\nu - g_{\mu\nu} P$ with $\boxed{\rho = \rho(t),\ P = P(t)}$

- FLRW:

$$(g_{\mu\nu}) = \mathrm{diag}\left(1, -\frac{R(t)^2}{1-kr^2}, -R(t)^2 r^2, -R^2(t) r^2 \sin^2\vartheta\right)$$

ansatz for solving field equations

- galaxies are freely falling: $x^i = \mathrm{const.}$

$$u^i = \frac{dx^i}{d\tau} = 0, \quad g_{00} = 1, \quad (u^\mu) = (u_\mu) = (c, 0, 0, 0)$$

$$\Rightarrow (T_{\mu\nu}) = \mathrm{diag}\left(c^2 \rho(t), +\frac{P(t) R^2(t)}{1-kr^2}, P(t) R^2(t) r^2, P(t) R^2(t) r^2 \sin^2\vartheta\right)$$

$$\Rightarrow T = \rho(t) c^2 - 3P(t)$$

4.1.2 Ordinary Differential Equations:

Inserting FLRW metric into Ricci tensor:

$$R_{00} = -3\frac{\ddot{R}(t)}{R(t)}$$

$$R_{11} = \frac{1}{1-kr^2}\left[R(t)\ddot{R}(t) + 2\dot{R}^2(t) + 2k\right]$$

$$R_{22} = r^2\left[R(t)\ddot{R}(t) + 2\dot{R}^2(t) + 2k\right]$$

$$R_{33} = +R_{22}\sin^2\vartheta$$

$\Rightarrow$ field equations:

1) temporal diagonal elements: $3\ddot{R}(t) - \Lambda R(t) = -\frac{4\pi G}{c^4}\left[\rho(t) c^2 + 3P(t)\right] R(t)$ (1)

2) spatial " " : $\dot{R}^2(t) + k - \frac{1}{3}\Lambda R^2(t) = \frac{8\pi G}{3c^2}\rho(t) R^2(t)$ (2)

independent of pressure P

$\frac{d}{dt}(2) - \frac{2}{3}\dot{R}(t)\,(1)$: $\quad \dot{\rho}(t) = -3\frac{\dot{R}(t)}{R(t)}\left[\rho(t) + \frac{P(t)}{c^2}\right] \qquad (3)$

3 quantities: $R(t)$, $\rho(t)$, $P(t)$ $\Longleftrightarrow$ 2 ordinary differential equations

One additional equation is missing: equation of state: $\quad P = P(\rho, \underbrace{t})$

depends on time

4.1.3 Equation of State: polytropic

generic ansatz: $\quad P(\rho) = K\rho^{\gamma}$

polytropic exponent

$K=0$ $\qquad\qquad$ $\gamma = 1$

incoherent matter: today $\qquad\qquad$ radiation dominance: 400.000 a after BB

$P \ll \rho c^2$: mass density is dominating $\qquad\qquad$ $P = \frac{c^2}{3}\rho$

$\Rightarrow \quad P = 0$

$\dot{\rho}(t) = -3\frac{\dot{R}(t)}{R(t)}\rho(t) \qquad\qquad \dot{\rho}(t) = -4\frac{\dot{R}(t)}{R(t)}\rho(t)$

$\Rightarrow \frac{d}{dt}\left[\rho(t)R^3(t)\right] = 0$

$\Rightarrow \rho(t)\cdot R^3(t) = \text{const.} \qquad\qquad \Rightarrow \rho(t)\,R^4(t) = \text{const.}$

mass conservation

Announcements:

1) Today, 15.45, 576: quantum droplets

2) Next lecture: 01.07. 12.15, 387 $\to$ 15.45, 576

3) Student talks: 9 talks $\to$ 08.07. 6 talks, 15.07. 3 talks