2.5 Supernova:

Nova: increase of luminosity by a factor $10^2 - 10^4$ for several hours
$\hat{=}$ appear to be a new star

Supernova: luminosity exceeds that of a nova by several orders of magnitude, $L > 10^9 L_\odot$ tends to be larger than luminosity of surrounding galaxy

example: supernova 1054, July 5, 1054 (Chinese astronomers)
occurred at Perseus arm of Milky Way, 6200 ly away from us
luminosity decayed after several months
gas envelope is ejected with a velocity of about 10^4 km/s

remark: on average there is one supernova each 10 – 30 years in Milky Way.

Phenomenologically one discriminates two types of supernova

	type I	type II
optical outburst of energy	$10^{-5} \dots 10^{-4} M_\odot c^2$	$10^{-6} \dots 10^{-5} M_\odot c^2$
ejected mass	$0.1 \dots 1 M_\odot$	$1 \dots 10 M_\odot$
spectra	no hydrogen lines	hydrogen lines

dramatic increase of observed light intensity comes from which energy source

mass defect: discussed in context of star equilibrium

M' : mass of all its constituents
M : gravitational mass

$$\Delta M = M' - M > 0 = -\frac{E_{grav.}}{c^2}$$

stability border: $R = \frac{9}{8} r_s$

$$\Rightarrow \frac{\Delta M}{M} = 0.64$$

10% of energy goes into the ejection of mass

Detailed description of a supernova goes beyond this lecture.

1) Type II: star of mass $M \sim 10 - - 30\, M_\odot$, metallic core

at some point fusion process stops, star cools down, thermal pressure is reduced, gravitational get larger, center gets pressed together

$\Rightarrow e^- + p \rightarrow n + \nu_e$

$\Rightarrow$ loss of Fermi pressure due to loss of electrons

$\Rightarrow$ implosion of inner part:

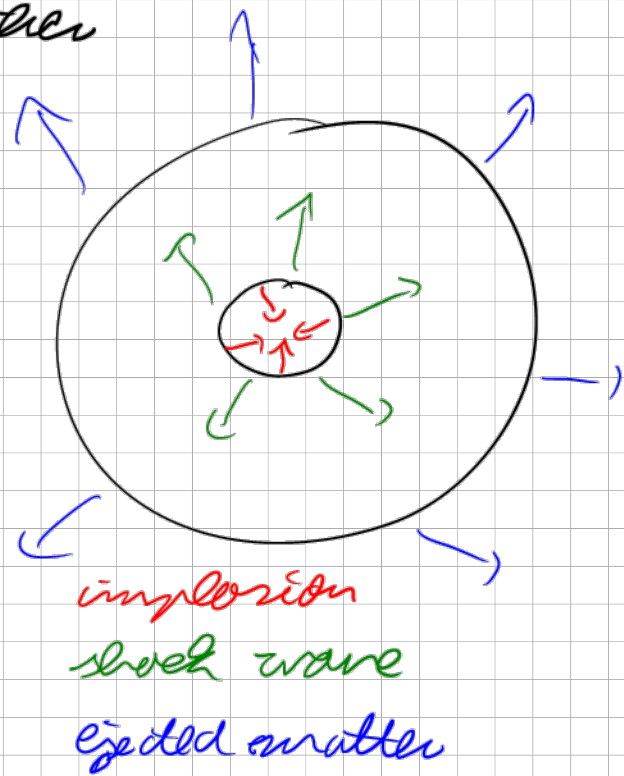
inner part gets pressed together

$\Rightarrow$ fusion to higher elements up to iron

but also leads to heavier elements

$\Rightarrow$ generation of heavier elements

$\Rightarrow$ shock wave: ejection of matter



Time of type II: 1054

today: crab nebula, extension of about 3 ly

=> remnant here: neutron star

Another example: SN 1987A (24.02.1987)

Large Magellanic Cloud, neighbouring galaxy of Milky Way, 160.000 ly from us

→ 19 neutrinos ("multi-messenger astronomy")

SN stems from a star $M = M_\odot 17$

=> no neutron star or black hole observed until now

2) Type I: based on white dwarf (helium) in a double star system

accretion of mass until Chandrasekhar mass which is the maximally possible mass of a white dwarf due to equilibrium

=> $e^- + p \rightarrow n + \nu_e$

Fermi pressure of electrons breaks down => gravitational collapse occurs

until $M \leq M_C \approx 1.3 M_\odot$

Fate: neutron star or black hole

Prime example: SN 199 D, occurred in galaxy NGC 4526, which is 60.000.000 ly away from us

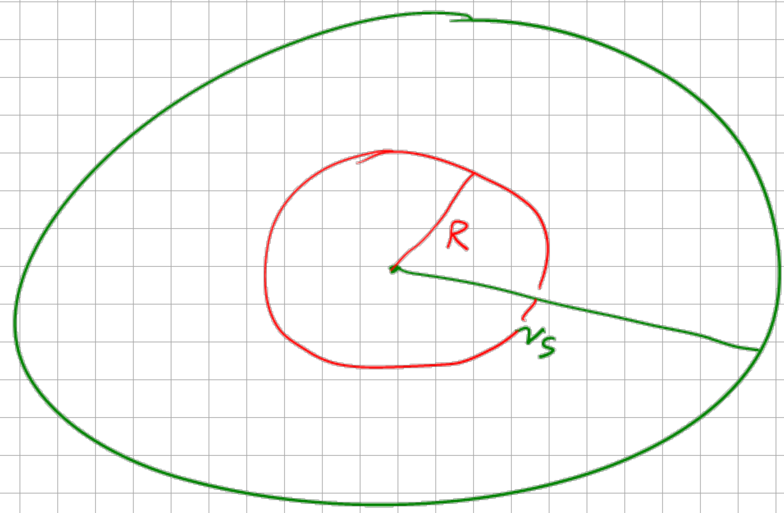
Type Ia supernovae turn out to be quite homogeneous

=> used in astronomy as distance measurement

=> list of observed supernovae

2.6 Black Hole: Physics Today 24 (1), 30 - 41 (1971)

2.6.1 Introduction:



$R < r_s = \frac{2GM}{c^2}$

black hole = star inside its own Schwarzschild radius

"black": no radiation from star surface gets outside the Schwarzschild radius

"hole": outside observer can not receive any information from it

Pierre-Simon Laplace (1749 – 1827):
flight velocity to escape a planet

$E_{kin} = \frac{m}{2} v_{flight}^2 = |E_{grav.}| = \frac{GMm}{R} \quad \Rightarrow \quad v_{flight} = \sqrt{\frac{2GM}{R}}$

photons $(m=0)$: $v_{flight} \Rightarrow c \quad \Rightarrow \quad R = \frac{2GM}{c^2} = r_s$

critical mass density for achieving a black hole (assume: no mass defect)

$R \approx r_s = \frac{2GM}{c^2}$

$M = \frac{4\pi}{3} \rho_{crit} R^3$

$\Bigg\} \quad M = \frac{4\pi}{3} \rho_{crit} \frac{8G^3 M^3}{c^6} \Rightarrow \rho_{crit} = \underbrace{\frac{3}{4\pi} \frac{M_\odot}{r_{s,\odot}^3}} \boxed{\left(\frac{M_\odot}{M}\right)^2}$

$= \frac{3}{4\pi} \frac{3 \cdot 10^{30}\,kg}{(3\,km)^3} = 1.8 \; 10^{19} \frac{kg}{m^3}$

largest densities: neutron star, nucleus $\Rightarrow \rho = 10^{17}\,kg/m^3$

$M = 10^8 M_\odot \quad \Rightarrow \quad \rho_{crit} = 2 \frac{g}{cm^3} \;\hat{=}\;$ density of water

$R = r_S \simeq \frac{2GM}{c^2} = 3\,\text{km}\,\frac{M}{M_\odot} \uparrow = 3\cdot 10^8\,\text{km} \quad (1\,\text{AU} = 1.5\cdot 10^8\,\text{km})$

$M = 10^8 M_\odot$

relativistic star cluster: Zeldovich and Podurets (1965)

2.6.2 Gravitational collapse

0 ⊢—— inside of star ——| star radius —— outside of star $r \geq r_0$ ——→ radial direction

Inside of star:

$$ds^2 = c^2 d\tau^2 - R^2(\tau)\cdot\left[\frac{dr'^2}{1-kr'^2} + r'^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right]$$

Gaussian normal coordinates adapted to free radial fall

Outside of star:

$$ds^2 = \left(1-\frac{r_s}{r}\right)c^2dt^2 - \frac{1}{1-\frac{r_s}{r}}dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$$

Schwarzschild metric: time dependence due to Birkhoff theorem

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star surface: $4\pi\, r_0'^2 R^2(\tau) = 4\pi\, r_0^2(t)$

$$r_0' R(\tau) = r_0(t)$$

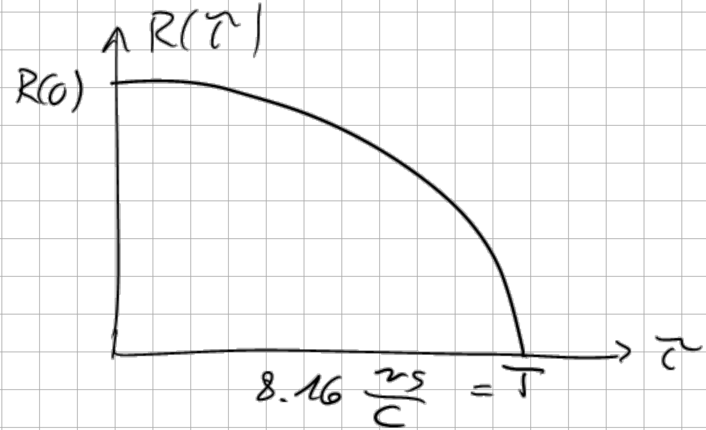
$r_0' \stackrel{!}{=} 1$ without loss of generality

initial condition: $R(0) = r_0(0)$, $\dot{R}(0) = 0$

$M_0 = \frac{4\pi}{3}\rho R(0)^3$, $\quad r_s = \frac{2GM}{c^2}$

$k \stackrel{\text{see earlier lecture}}{=} \frac{2GM}{c^2}\frac{1}{R(0)} = \frac{r_s}{R(0)} \underset{R(0)=3r_s}{=} \frac{1}{3} < 1$ $\quad$ ($M_0 = M$)

dimensionless parameter



$$\frac{r_s}{c} = \frac{2GM_\odot}{c^3}\left(\frac{M}{M_\odot}\right) = 10^{-5}\,s\left(\frac{M}{M_\odot}\right)$$