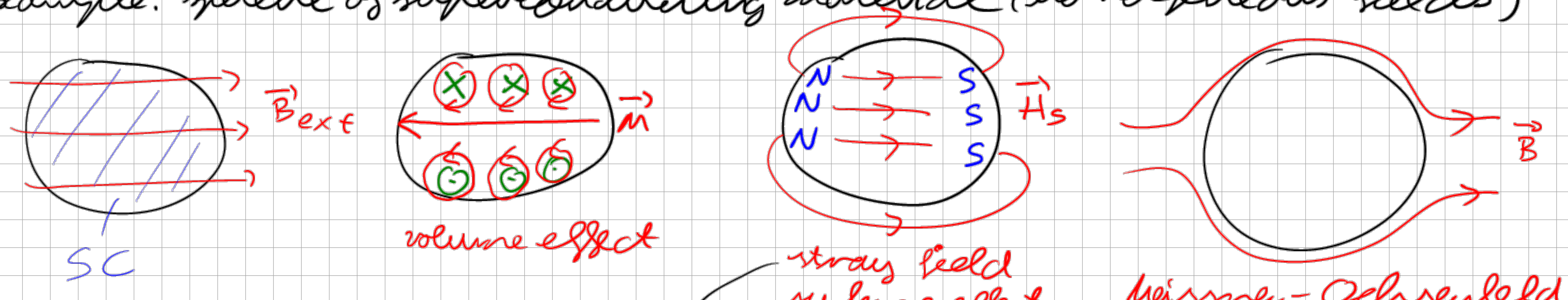
2.4 Description of Complete Diamagnetism:

How does the probe shape affect magnetization curves of type I superconductors?

2.4.1 Physical Picture:

Example: sphere of superconducting material (homogeneous fields)



$\hat{=}$ demagnetization field

2.4.2 Stray Field:

$$\vec{B}_{int} = \mu_0 (\vec{M} + \vec{H}_s)$$

$$\vec{H}_s = - \underset{\sim}{N} \, \vec{M}$$

demagnetization tensor

$$N_{ij}\cancel{(\vec{r})} = \frac{-1}{4\pi} \oint_{\partial V} dF_j' \frac{\partial}{\partial x_i'} \frac{1}{|\vec{r} - \vec{r}'|}$$

for special probe this $\vec{r}$-dependence is dropped $\Rightarrow$ homogeneous stray field

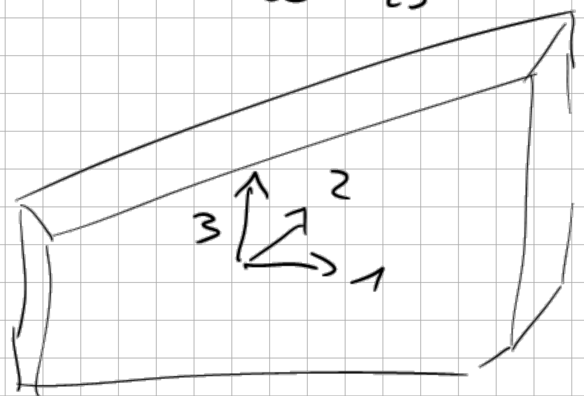
$$\mathrm{Tr}\, N = N_{ii} \overset{\text{Gauß}}{\underset{\text{law}}{=}} \frac{-1}{4\pi}\int_V dV' \underbrace{\Delta' \frac{1}{|\vec{r}' - \vec{r}''|}}_{= -4\pi\, \delta(\vec{r}' - \vec{r}'')} \overset{\vec{r} \in V}{=} 1$$

This and symmetry arguments allow to determine N_{ij}

1) infinitely thin layer

$N_2 = N_3 = 0$ (no surface effects at infinity)

$N_1 + N_2 + N_3 = 1 \Rightarrow N_1 = 1$



$$N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2) infinitely long cylinder:

$$N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$

3) sphere:

$$N = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$$

Consequences:

$$B = \mu_0 \Big(\underset{\substack{\uparrow \\ \text{external}}}{H} + \underset{\substack{\uparrow \\ \text{volume}}}{M} \overbrace{- N M}^{= H_S} \Big) \overset{!}{=} 0 \Rightarrow M(H) = -\frac{1}{1-N} H$$

surface

magnetic susceptibility:

$$\chi = \frac{\partial M(H)}{\partial H} = -\frac{1}{1-N}$$

$0 \leq N \leq 1 \Rightarrow -\infty < \chi \leq -1$

Another (formal ?) point of view:

$\boxed{B_{eff}} = \mu_0 (H + H_S) = \mu_0 (H - NM) = \boxed{\mu_0 \frac{1}{1-N} H} = -\mu_0 M$

sphere: $N = \frac{1}{3} \Rightarrow B_{eff}(H) = \mu_0 \underbrace{\frac{3}{2}}_{\text{enhancement}} H$

$\chi_{eff} = \frac{\partial M}{\partial B_{eff}} \mu_0 = -1 \mathrel{\hat{=}}$ ideal diamagnetism

2.4.3 Intermediate State:

Consistency problem:

$B_{eff}(H_c) = \mu_0 H_c = \mu_0 \frac{1}{1-N} H_c \Rightarrow 1 \overset{?}{=} \frac{1}{1-N}$

$\Rightarrow$ homogeneous superconductor not possible

But also homogeneous normalconductor not possible due to $\chi = 0$

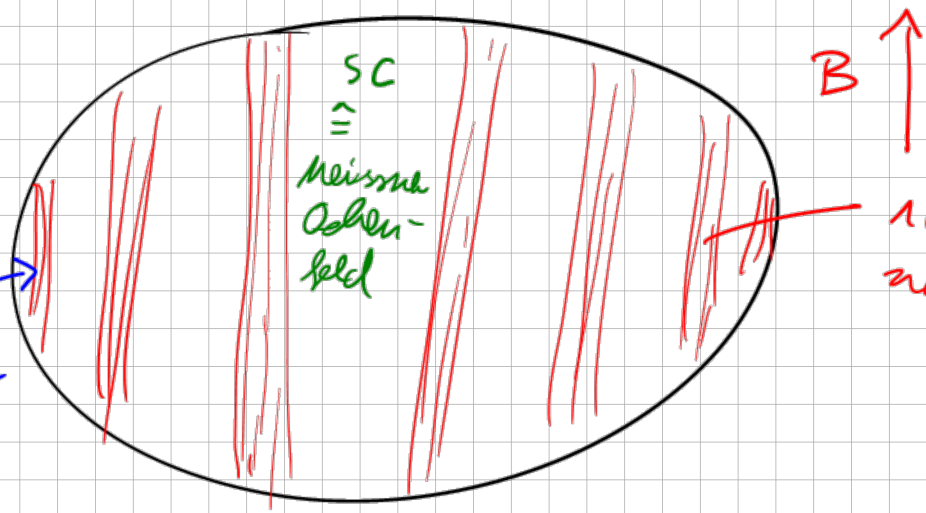
Dilemma solved in 1936 by Peierls and London

$\Rightarrow$ inhomogeneous intermediate state

superconductor is divided into normal conducting and superconducting regions

$B_{eff} = B_c = \mu_0 H_c$

normal conducting regions start at equator to minimiz sc – nc areas



10^5 flux quanta normal conducting

normal cond. region	$B_n = \mu_0 H \geq H_c \mu_0$	$M_n = 0$
supercond. region	$B_s = 0$	$M_s = -\frac{1}{1-N} H$

Needed: spatial average of magnetization and magnetic conduction

1) Border of criticality

$$B_{eff}(H_{cI}) = \mu_0 H_c \quad \text{with} \quad B_{eff}(H) = \mu_0 \frac{1}{1-N} H \Rightarrow H_{cI} = \underbrace{(1-N) H_c}_{\leq H_c}$$

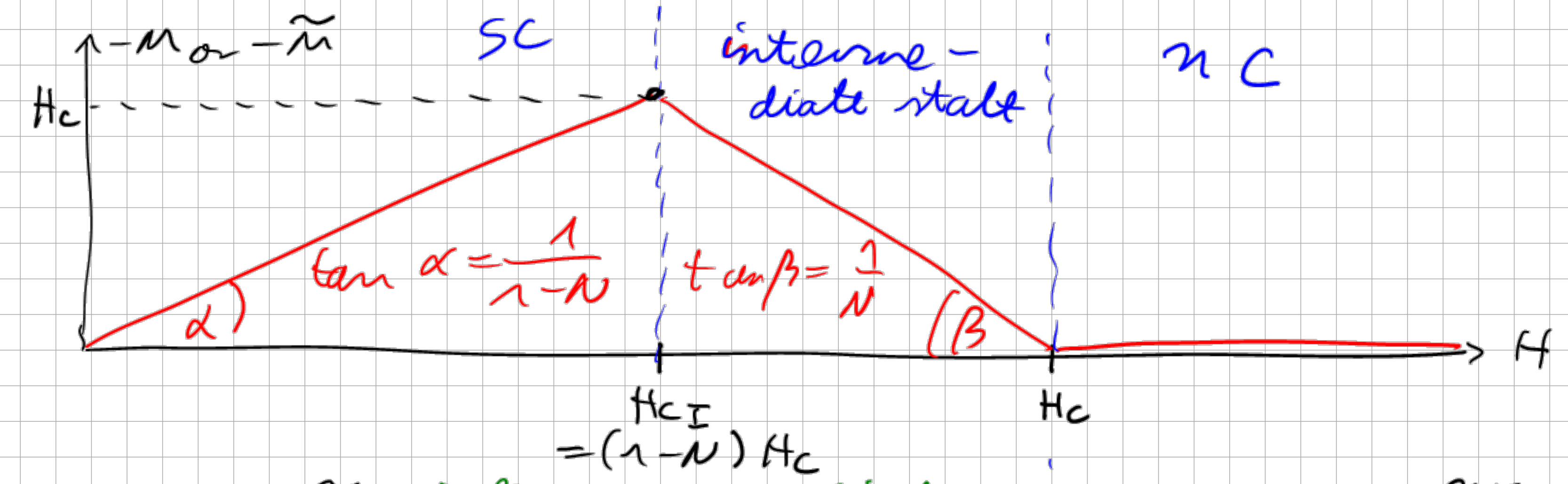
$$M_{cI} = -\frac{1}{1-N} H_{cI} \stackrel{\wedge}{=} -H_c \checkmark$$

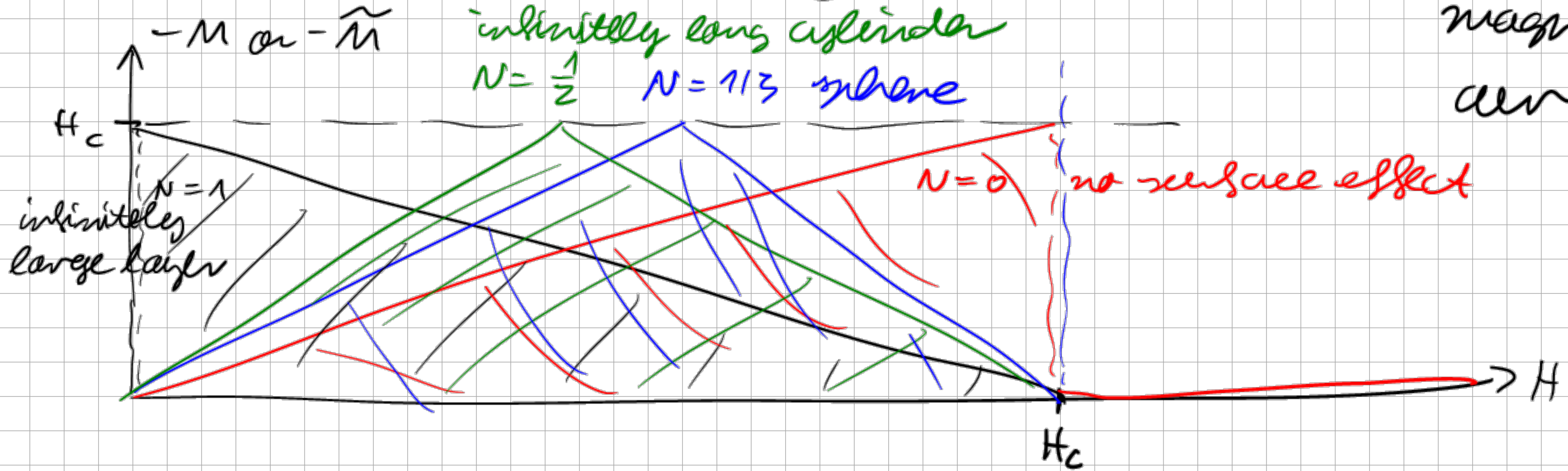
2) spatial averages

$$\tilde{B}_{eff}(H) = \mu_0 (H - N \tilde{M})$$

Characterization of intermediate state: $\tilde{B}_{eff}(H) = \mu_0 H_c$ $\Big\}$ $\tilde{M}(H) = \frac{H - H_c}{N}$

$$\tilde{M}(H_{cI}) = \ldots = -H_c \checkmark, \quad \tilde{M}(H_c) = 0$$





magnetization curves

Observation: area under all demagnetization curves are equal

$\hat{=}$ energy

$$E_n - E_s = -\mu_0 V \int M(H)\, dH = \frac{\mu_0}{2} V H_c^2 = \frac{B_c^2}{2\mu_0} V > 0$$

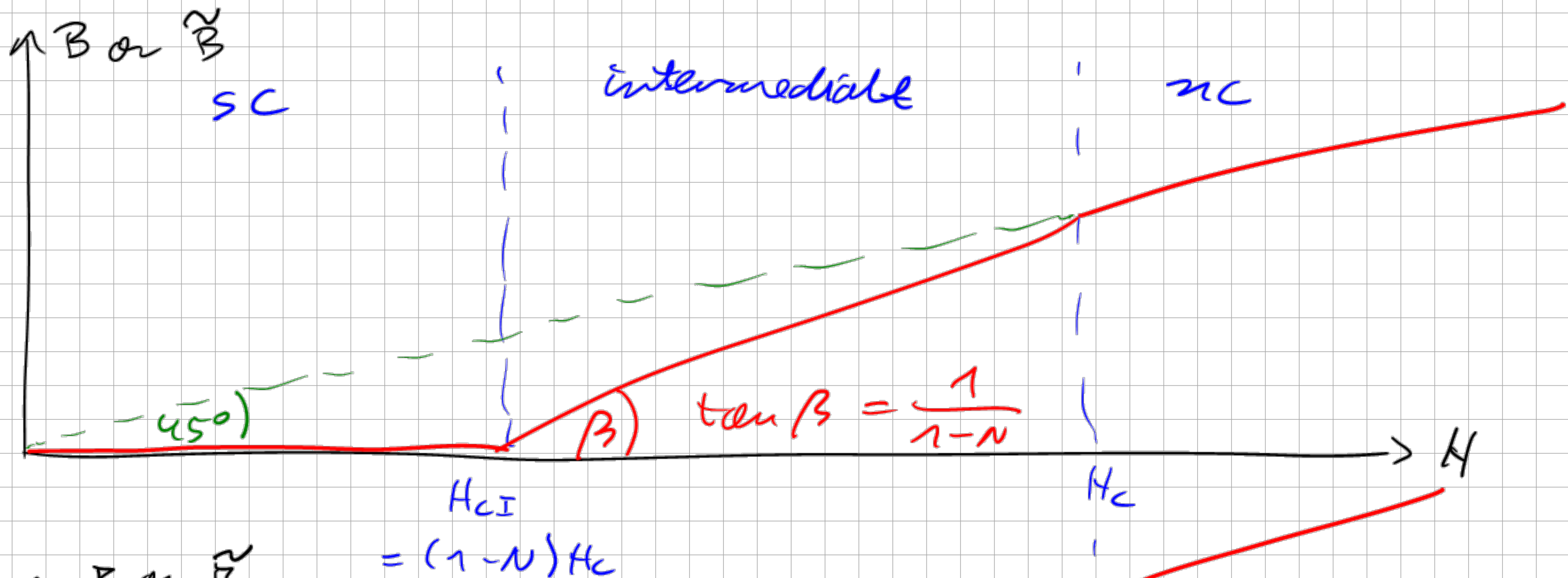
$\Rightarrow$ superconductor has lower energy than normal conductor

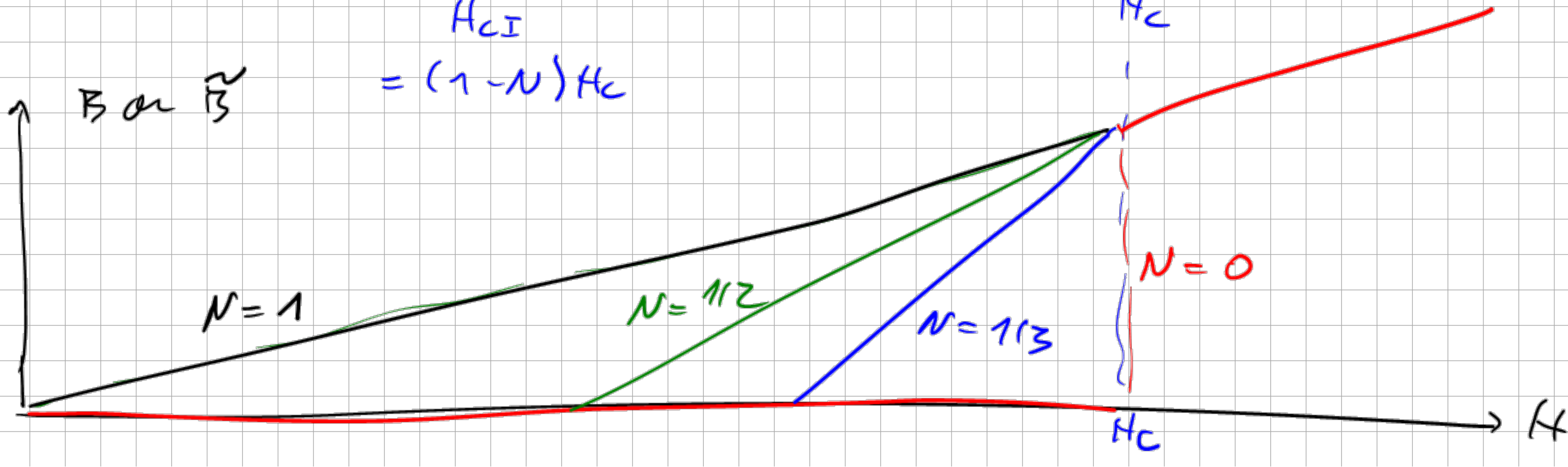
$\hat{=}$ energy gain is due to pairing of electrons near Fermi surface corresponding magnetization curves:

$$\tilde{B} = \mu_0 \left(H + (1-N) \underbrace{\tilde{m}}_{\frac{H - H_c}{N}} \right) = \mu_0 \left[\frac{1}{N} H + \left(1 - \frac{1}{N}\right) H_c \right]$$

Onset of criticality: $\tilde{B}(H_{cI}) \equiv 0$ (sc ends)

$\tilde{B}(H_c) = \mu_0 H_c$ (int. state ends

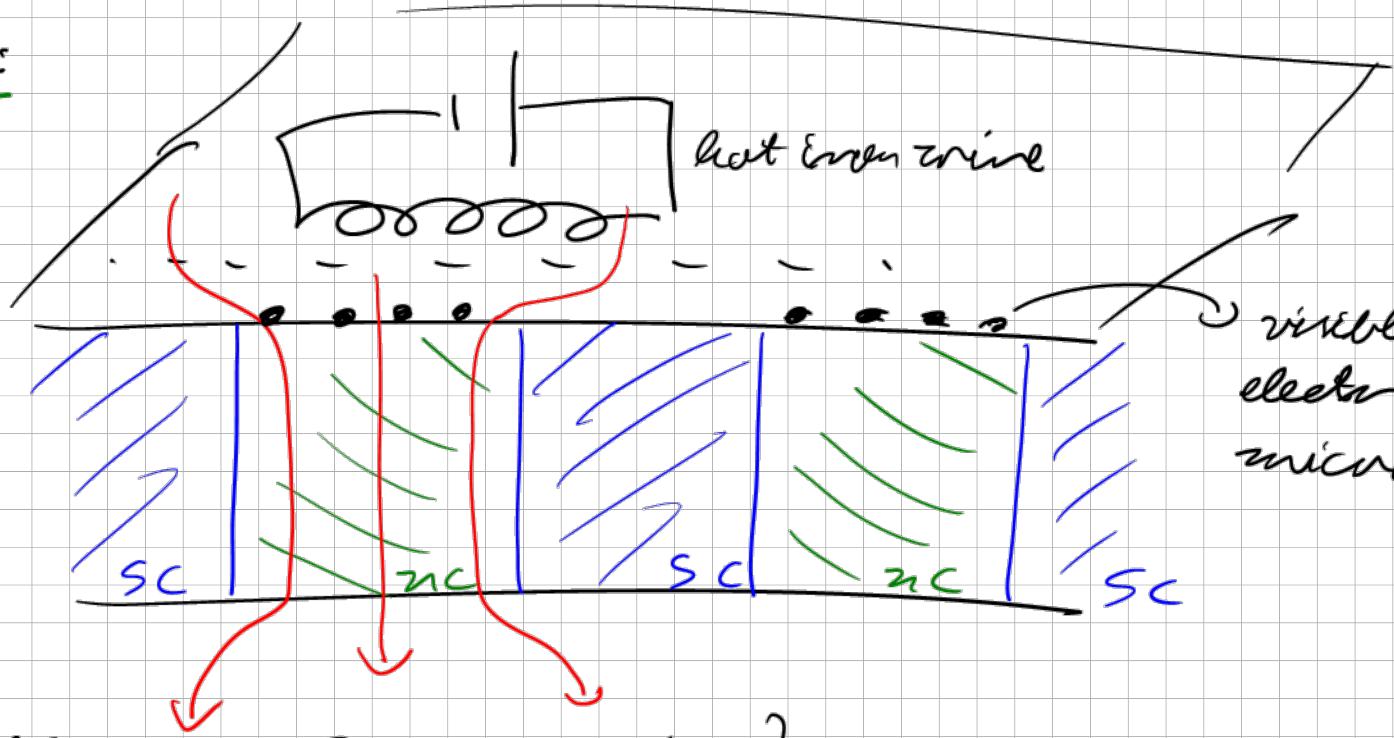




2.5 Experiments:

2.5.1 Iron Colloids:

iron colloids with diameter of 50 nm



- intermediate state of type I SC → 1966
- Shubnikov phase of type II SC → 1968

} MPI-FKF, Stuttgart, Eßmann and Träuble

2.5.2 Niobium Powder:

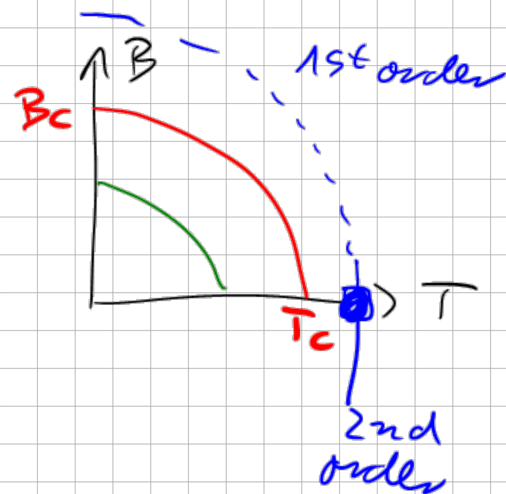
How can we make SC regions visible? SC powder

Which requirements do powder and probe have?

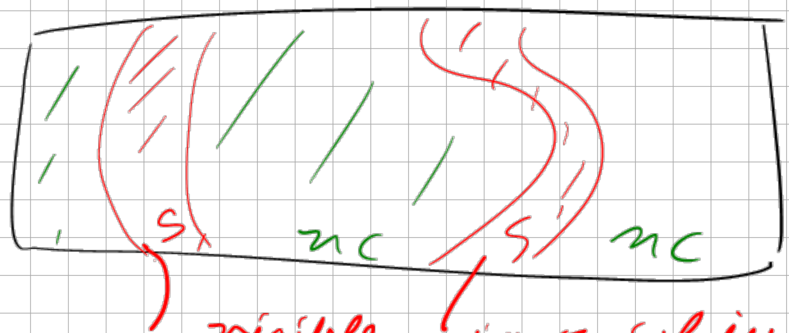
$B_c(\text{probe}) < B_c(\text{powder})$

$T_c(\text{probe}) < T_c(\text{powder})$

→ KOK formula



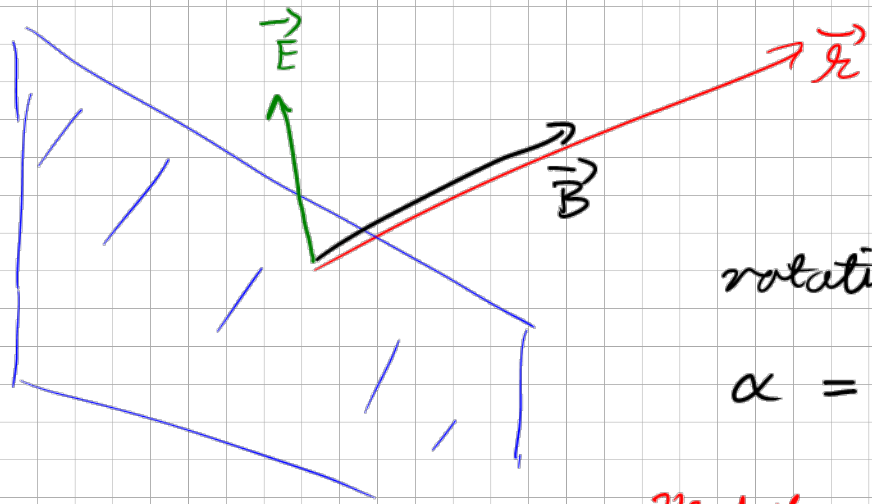
niobium represents an ideal diamagnet is expelled from regions of large magnetic fluxes, i.e. n s region



visible via niobium powder

2.5.3 Faraday effect: magneto-optical phenomenon

$\hat{=}$ interaction between light and magnetic properties



rotating of polarization detectable via polarizer

rotation angle:

$$\alpha = \underset{\uparrow}{V} \; \underbrace{d}_{\text{material width}} \; B$$

Verdet constant

$$= \frac{-e}{m} \; \frac{\lambda}{2c} \; \underbrace{\frac{dn}{d\lambda}}_{\text{dispersion}}$$

$\leftarrow$ index of refraction

Faraday effect, i. e. α, largest near an absorption line

electrons oscillate resonantly, in presence of B electrons feel a Lorentz force → changes polarization

$\Rightarrow \omega_L = \dfrac{eB}{2m}$ Larmor frequency $\Rightarrow \alpha = \omega_L \, d \, \dfrac{\lambda}{c} \dfrac{dn}{d\lambda}$