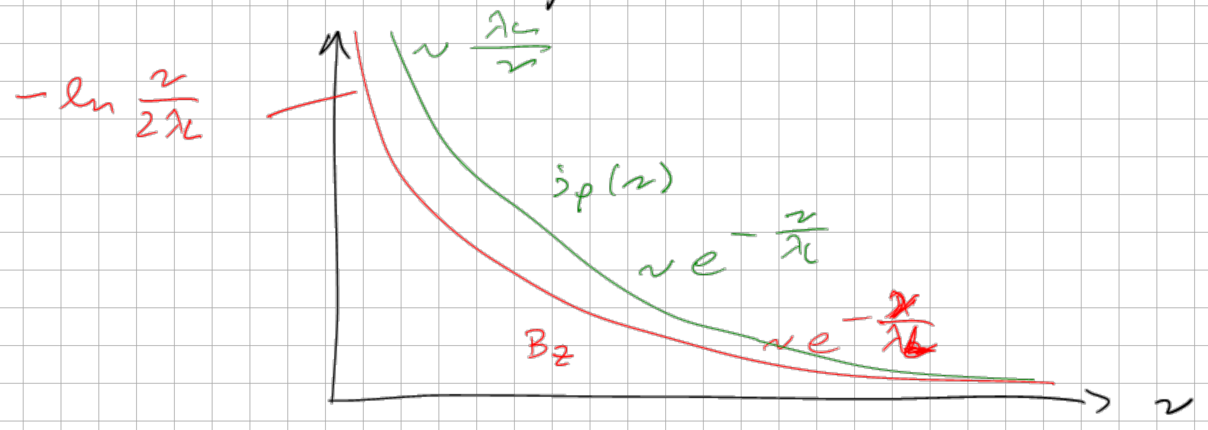
Magnetic Induction $\vec{B}$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ near a flux quantum

Superconducting current $\vec{j}_s$



4.9 Vector Potential:

reminder: $\quad \text{rot}(\Lambda_s \vec{j}_s) + \vec{B} = \vec{0}\,, \quad \text{div}\,\vec{B} = 0 \;\Rightarrow\; \vec{B} = \text{rot}\,\vec{A}$

$$\Rightarrow \text{rot}\left(\Lambda_s \vec{j}_s + \vec{A}\right) = \vec{0} \quad \Rightarrow \quad \vec{A} = -\Lambda_s \vec{j}_s$$

in a single connected region

multiple connected region, like nearby a vortex:

$$\vec{A} + \Lambda_s \vec{j}_s = \text{grad}\,\underbrace{\chi}_{\text{gauge function}} \quad (*)$$

fix gauge via gauge condition: steady state $\rightarrow$ $\text{div}\,\vec{j}_s = 0$

$$\text{div}(*):\quad \left.\begin{matrix} \underbrace{\text{div}\,\vec{A}}_{=\;\text{Coulomb gauge}} = \Delta\chi \end{matrix}\right\} \Rightarrow \Delta\chi = 0$$

cylinder-symmetry: $\Delta \chi = \frac{\partial^2}{\partial \varphi^2} \chi(\varphi) = 0 \rightarrow \chi(\varphi) = k\varphi + c$

$\text{grad}\, \chi = \frac{1}{r} \frac{\partial \chi}{\partial \varphi} \vec{e}_\varphi = \frac{k}{r} \vec{e}_\varphi$ k is not yet fixed

quantization condition

$$\oint_{\partial F} (\vec{A} + \Lambda_s \vec{j}_s) \cdot d\vec{r} \overset{!}{=} \phi_0 \Rightarrow \frac{k}{\cancel{R}} 2\pi \cancel{R} \overset{!}{=} \phi_0 \Rightarrow k = \frac{\phi_0}{2\pi}$$

circle of radius R

Evaluate (*) in cylinder coordinates: $A_\varphi(r) = -\Lambda_s \underbrace{j_{s\varphi}(r)}_{\text{last current}} + \frac{k}{r}$

$$\Rightarrow A_\varphi(r) = \frac{\phi_0}{2\pi} \left\{ \underbrace{\frac{1}{r}}_{\text{gauge function}} - \underbrace{\frac{1}{\lambda_L} K_1\left(\frac{r}{\lambda_L}\right)}_{\sim \frac{\lambda_L}{r} \text{ for } r \to 0} \right\} \xrightarrow{r \to 0} 0$$

4.10 Extension of London Theory:

- penetration length λ_L
- near flux quantum divergency of $\vec{B}$
  origin: assumption of spatial homogeneity of n_s is wrong
- Ginzburg–Landau theory: macroscopic wave function Ψ
  $\rightarrow n_s = |\Psi|^2$
- probability current $\rightarrow$ superconducting current density

$$\vec{j}_s = \frac{e_s \hbar}{2 i m_s} (\Psi^* \vec{\nabla} \Psi - \Psi \vec{\nabla} \Psi^*) - \frac{e_s^2}{m_s} \vec{A} \Psi^* \Psi$$

points to G.–L. theory

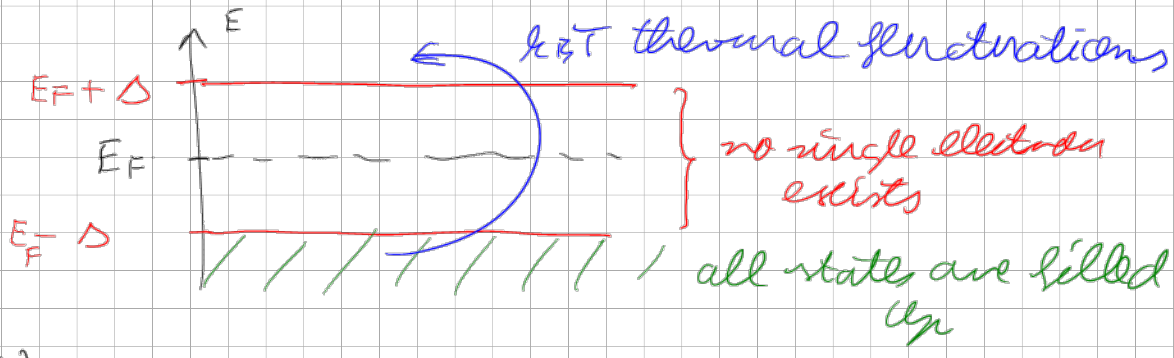
$\hat{=}$ London theory

no inhomogeneities: $\vec{j}_s \to -\frac{e_s^2}{m_s} \vec{A}\, n_s$ in simply connected region

- Heuristic estimate for the change of ψ or n_s: coherence length ξ
- Thermodynamic discussion:

$$E = \frac{p^2}{2m} \Rightarrow \Delta = \Delta E = \frac{p}{m}\Delta p = v_F \Delta p$$

$$\Delta p \Delta x \geq \frac{\hbar}{2} \Rightarrow \Delta x \geq \frac{\hbar}{2\Delta p} = \frac{\hbar v_F}{2\Delta} = \xi$$



- Hallmark of BCS theory:

$$2\Delta(0) = 3.52\, k_B T_c \longrightarrow \xi = 0.18\,\frac{\hbar v_F}{k_B T_c}$$

element	v_F	T_c	ξ	λ_L
Al	$1.2\cdot 10^6$ m/s	1.2 K	10^4 Å	300 Å

$$\varkappa = \frac{\lambda_L}{\xi} = 0.03 < \varkappa_c = 0.7 \Rightarrow \text{type I SC}$$

Changes of length scale due to n_s:

1) $\lambda_L \sim \frac{1}{\sqrt{n_s}}$

2) $\xi \sim v_F \sim n_s^{1/3}$ (neglect T_c or Δ depend on n_s)

Chapter 5: Ginzburg-Landau

Historic Remark:

Ginzburg / Landau 1950 (e_s, m_s as model parameters)

BCS theory 1957 → Cooper pairs

Gorkov 1959 → derivation of Ginzburg-Landau ($e_s = 2e$, $m_s = 2m$)

Abrikosov flux lines of type II superconductors

⇒ GLAG theory

1962 2003

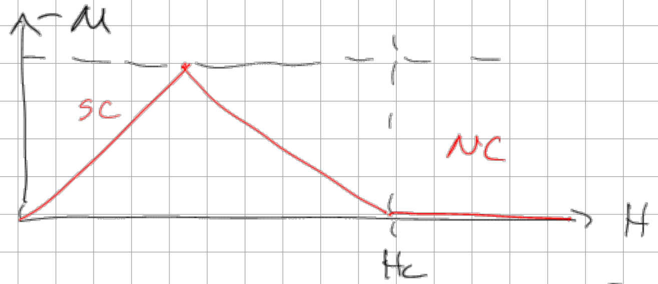
5.1 Motivation: GL beyond London theory?

5.1.1 Flux Quantum:

$\vec{B}$, $\vec{j}_s$ near flux quantum diverge → has to be cured.

5.1.2 Magnetization Curves:

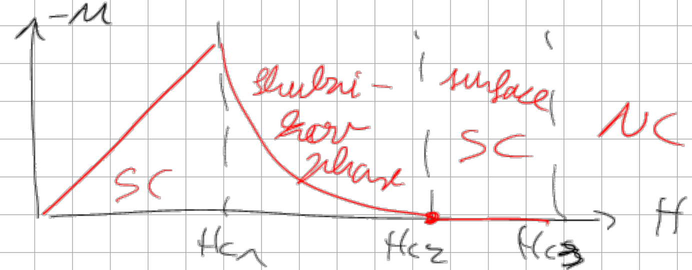
type I



$\kappa = \frac{\lambda_L}{\xi}$

$\kappa_c \approx 0.7$

type II



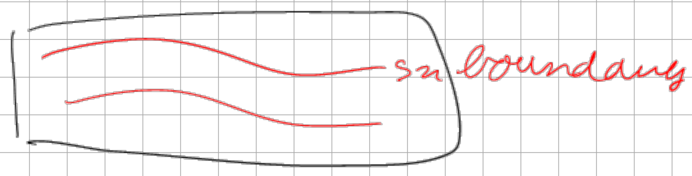
- critical fields?
- magnetization curves?

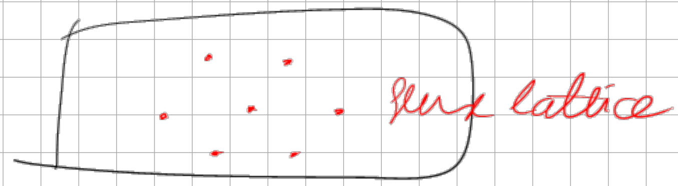
} applications

5.1.3 Flux Penetration:

type I

type II



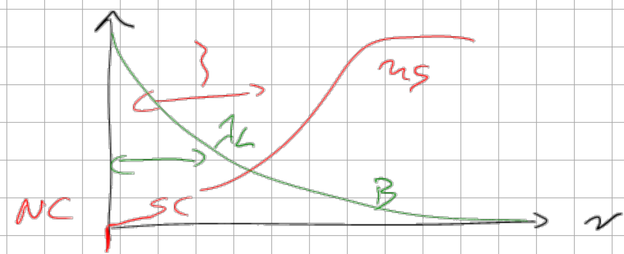


millions of flux quanta penetrate macroscopically SC

single flux quanta arise, forming hexagonal lattice

5.1.3 surface energy:

Two energy contributions at surface of SC



1) On length scale ξ cooper pairs have to be broken

→ loss of energy

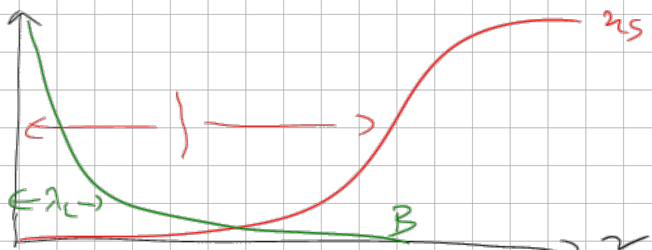
$$\gamma_1 = \left(\frac{E}{A}\right)_1 = \xi \, \frac{B_c^2}{2\mu} > 0$$

2) SC has lower energy than NC, see thermodynamics

$$\gamma_2 = \left(\frac{E}{A}\right)_2 = -\lambda_L \, \frac{B_c^2}{2\mu} < 0$$

$$\Rightarrow \gamma = \gamma_1 + \gamma_2 = (\xi - \lambda_L)\,\frac{B_c^2}{2\mu} \begin{cases} > 0 & ; \ \xi > \lambda_L \\ < 0 & ; \ \xi < \lambda_L \end{cases}$$

Type I ($\varkappa = \frac{\lambda_L}{\xi} < 1$)



Type II ($\varkappa = \frac{\lambda_L}{\xi} > 1$)

$\gamma > 0$	$\gamma < 0$
homogeneous SC state	inhomogeneous SC state
=> only small sn boundaries formed	many flux quanta are preferred

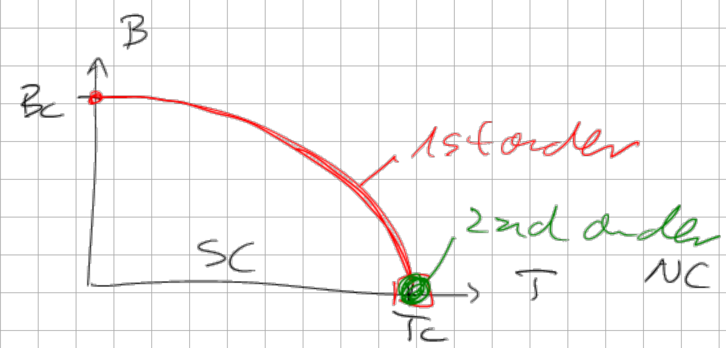
5.1.5 Ginzburg-Landau parameter: $\varkappa = \frac{\lambda_L}{\xi}$

a detailed analysis: $\varkappa_c \neq 1 \longrightarrow \varkappa_c = \frac{1}{\sqrt{2}} \approx 0.7$

material	Al	Nb_3Sn	high T_c S
$\varkappa$	0.03	30 – 100	several hundreds

type I (Al); type II (Nb_3Sn, high T_c S)

5.2 Homogeneous SC: (Landau) type I



2nd order phase transition

universality principle of critical phenomena:

- spatial dimension
- order parameter dimension

5.2.1 Postulates:

1) Complex wave function $\Psi = \Psi(T) \hat{=}$ order parameter

SC phase: $\Psi(T < T_c) \neq 0$

NC phase: $\Psi(T > T_c) = 0$

2) Physical interpretation: $|\Psi(T)|^2 = n_s$

3) V changes of free enthalpy ignored

$\Rightarrow G_s = G_s(\Psi, T)$

4) Near T_c: Ψ is small (continuous phase transition)

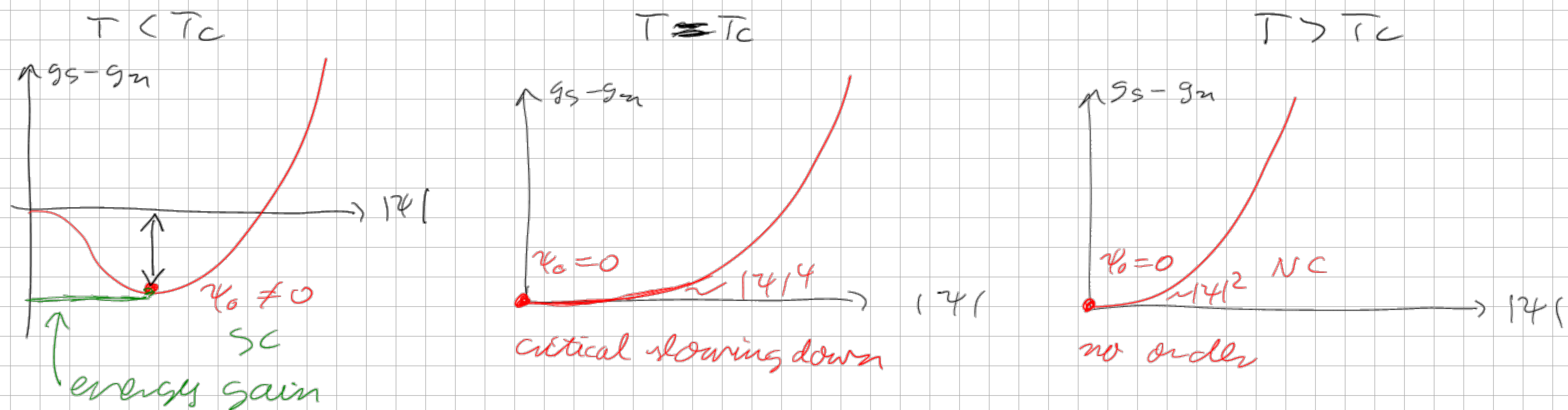
$$g_s = \frac{G_s}{V} = g_n(T) + \alpha(T)|\Psi|^2 + \frac{1}{2}\beta(T)|\Psi|^4 + \dots$$

5) $\alpha(T) = (T - T_c)\,\alpha_c'$, $\alpha_c' = \left.\frac{d\alpha(T)}{dT}\right|_{T_c} > 0$

$\beta(T) = \beta > 0$

$\Rightarrow$ 2nd order phase transition

5.2.2 Qualitative discussion:



5.2.3 Equilibrium states:

Extremise free enthalpy: $\frac{\partial g_s}{\partial \Psi^*} = 0$

$$\Rightarrow \quad \psi_0 \{ \alpha(T) + \beta(T)|\psi|^2 + \dots \} = 0$$

1) $\psi_0 = 0$ for all T

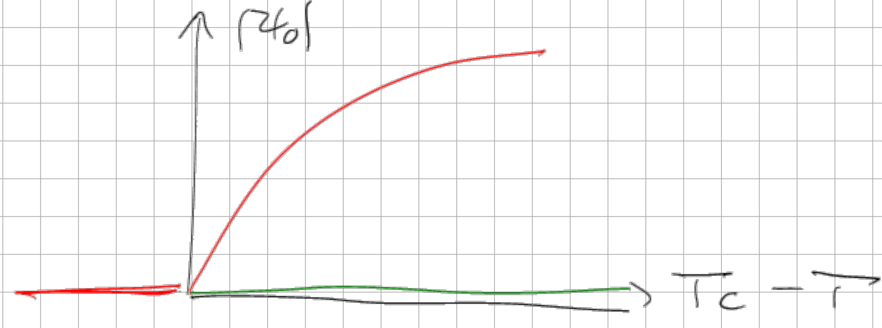
2) $|\psi_0| = \sqrt{-\frac{\alpha(T)}{\beta}}$ for $T < T_c$

5.2.4 Stability of Equilibrium States:

Minima of equilibrium states

1) $g_s(\psi_0 = 0) = g_n$ for all T

2) $g_s(\psi_0 \neq 0) = g_n - \frac{\alpha(T)^2}{2\beta}$ for $T < T_c$



Discussion:

1) Symmetry breaking

$\psi, \psi^* \longrightarrow \psi e^{i\varphi}, \ \psi^* e^{-i\varphi}$: $U(1)$ symmetry g_s

$T < T_c$: $\psi_0 = |\psi_0| e^{i\varphi_0}$ which no longer has $U(1)$-symmetry

2) Critical slowing down near T_c

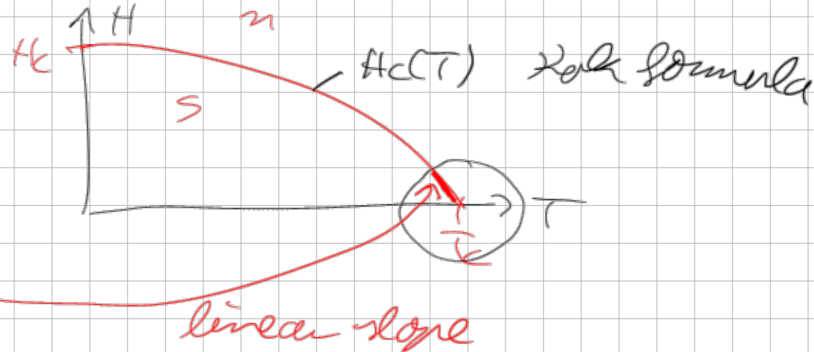
5.2.5 Condensation energy density:

$T < T_c$: energy gain

$$\Delta g = g_n - g_s = \frac{\alpha(T)^2}{2\beta}$$ — Landau parameters

depends on Landau parameters

$$\Delta g = g_n - g_s = \frac{\mu_0}{2} H_c(T)^2$$ experimentally accessible



$$\frac{\alpha(T)^2}{2\beta} = \frac{\mu_0}{2} H_c(T)^2 \Rightarrow H_c(T) = \frac{\alpha_c'}{\sqrt{\beta\mu_0}} \cdot (T_c - T)$$

In accordance with Kohl formula:

$$H_c(T) = H_c(0) \cdot \left(1 - \frac{T^2}{T_c^2}\right) = H_c(0) \frac{(T_c - T)(T_c + T)}{T_c^2} \overset{T \approx T_c}{\approx} \frac{2H_c(0)}{T_c^2}(T_c - T)$$

Landau parameters $\alpha_c', \beta \longleftrightarrow H_c(0), T_c$

5.2.6 Remarks:

1) G-L theory is not microscopic, instead it contains material parameters $\alpha(T), \beta$

2) $n_s = |\Psi|^2 = -\frac{\alpha(T)}{\beta} = \frac{\alpha_c'}{\beta}(T_c - T)$

$$\Rightarrow \lambda_L \sim \frac{1}{\sqrt{n_s}} \sim \frac{1}{\sqrt{T_c - T}} \sim (T_c - T)^{-\nu} \Rightarrow \nu = 0.5$$ (MF exponent

thermal fluctuations: $\nu = 0.63$