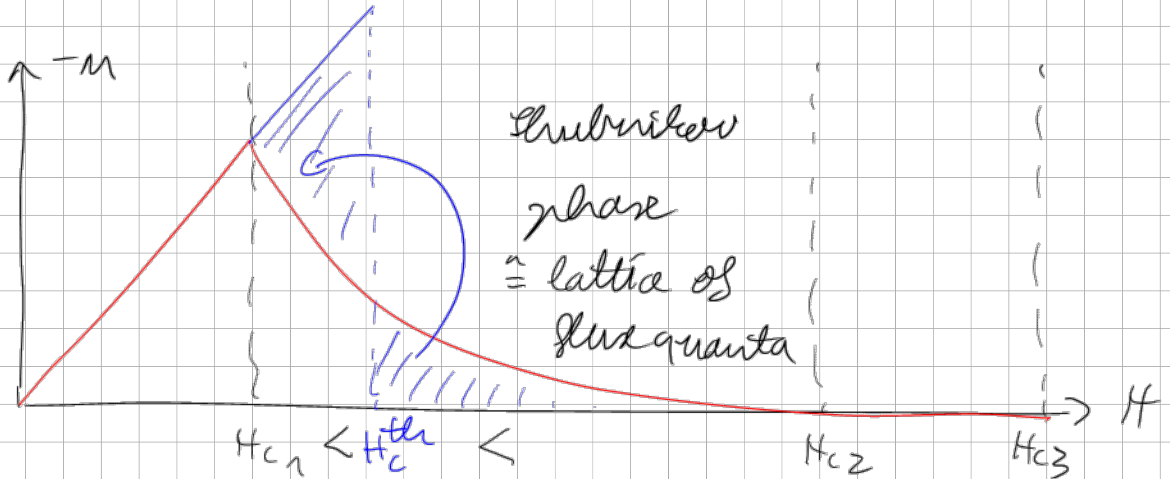


$$\underbrace{-\mu_0 \int_0^{H_{c2}} dH\, M(H) = \frac{\mu_0}{2} H_c^{th\,2} = \frac{B_c^{th\,2}}{2\mu_0}}_{} = \frac{\alpha^2}{2\beta} \Rightarrow B_c^{th} = \sqrt{\frac{\mu_0}{\beta}}(-\alpha), \quad \alpha < 0$$

magnetic energy per volume stored in Meissner-Ochsenfeld effect due to London theory

superconductor energy stored per volume in the condensation of Cooper pairs due to Landau theory

Upper critical field H_{c2}

linearization of Ginzburg-Landau theory:

1) $\Psi \ll \Psi_0 = \sqrt{-\frac{\alpha}{\beta}}$

2) $\vec{A}(\vec{r}) = B_z \begin{pmatrix} 0 \\ x \\ 0 \end{pmatrix}$ Coulomb gauge $\hat{=}$ $\vec{B}(\vec{r}) = \mathrm{rot}\,\vec{A}(\vec{r}) = B_z \vec{e}_z$

$\Rightarrow$ Schrödinger equation for harmonic oscillator

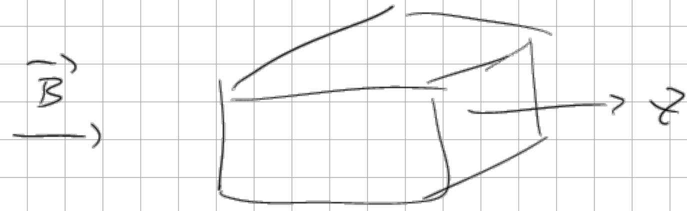
eigen-values $\quad \alpha_n = \hbar \omega_0 (n + \frac{1}{2})$, $\quad \alpha' = \alpha + \frac{\hbar^2 \delta^2}{2 m_s}$, $\delta^2 \rightarrow k_z$, $\quad \omega_0 = \frac{e_s B_z}{m_s} = \frac{e B_z}{m}$ cyclotron frequency

$\sim B_z$

eigen-functions $\quad \Psi_n(x') = N_n H_n\left(\sqrt{\frac{m_s \omega_0}{\hbar}} x'\right) e^{-\frac{m \omega_0}{2\hbar} x'^2}$

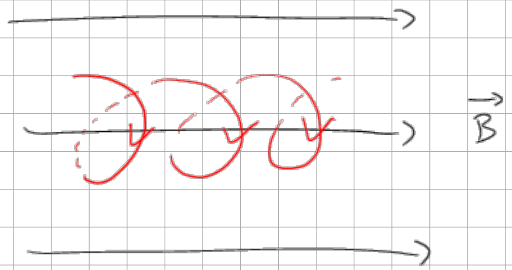
$x' = x - \frac{\hbar \gamma}{e_s B_z}$

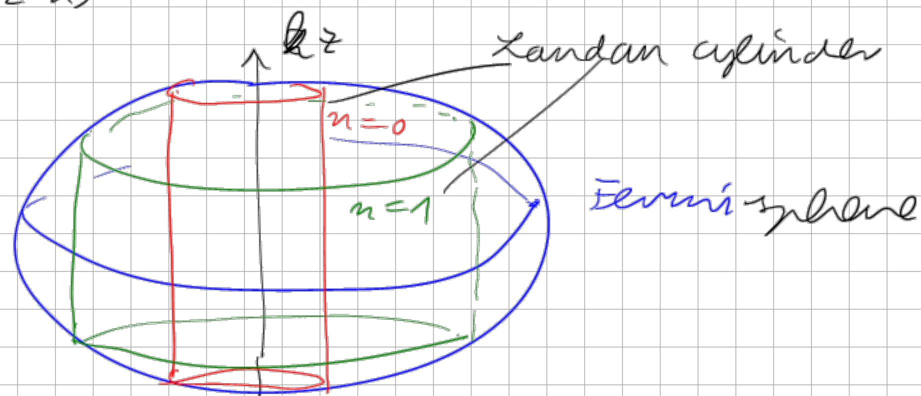
6.2.3 Quantum Mechanics:



$$E_n = -\alpha_n' = \underbrace{\hbar\omega_0\left(n+\frac{1}{2}\right)}_{\text{circular motion of electrons in xy-plane}} + \underbrace{\frac{\hbar^2\delta^2}{2m_s}}_{\text{free motion in z-direction}}$$

$$E_n = -\vec{m}\cdot\vec{B} + \frac{\hbar^2\delta^2}{2m_s}, \qquad m_z = -\frac{\hbar e_s}{2m_s} \quad \text{Bohr magneton}$$





$$\frac{\hbar^2}{2m_s}\left(k_x^2+k_y^2\right) = \hbar\omega_0\left(n+\frac{1}{2}\right)$$

6.2.4 Upper Critical Magnetic Field:

QM: magnetic field given $\longrightarrow$ energy eigenvalues

GL: given quantized energies $\longrightarrow$ upper critical field

$$B_z(n,\delta) = \frac{2m_s}{\hbar e_s}\left(-\alpha - \frac{\hbar^2\delta^2}{2m_s}\right)\frac{1}{2n+1}$$

$$B_{c2} = \max_{\substack{n\in\mathbb{N}_0\\ \delta\in\mathbb{R}}} B_z(n,\delta) = B_z(n=0,\delta=0) = \frac{2m_s}{\hbar e_s}(-\alpha), \quad \alpha = \alpha_c'(T-T_c) < 0$$

And other $B_z(r,\delta)$ have no physical meaning.

6.2.5 Consequences:

$$\frac{B_{c2}}{B_c^{th}} = \sqrt{\frac{\beta}{\mu_0}}\;\frac{2 m_s}{\hbar e_s} \quad \Rightarrow \text{ not } T \text{ dependent, but material dependent due to } \beta$$

$\longrightarrow$ What is the physical interpretation of this dimensionless ratio?

$$\left.\begin{aligned} \lambda_L &= \sqrt{\frac{m_s \beta}{e_s^2 \mu_0 \alpha'(T_c - T)}} \sim \frac{1}{\sqrt{T_c - T}} \\ \xi &= \sqrt{\frac{\hbar^2}{2 m_s (T_c - T)\alpha'}} \sim \frac{1}{\sqrt{T_c - T}} \end{aligned}\right\}$$

Ginzburg-Landau parameter

$$\kappa = \frac{\lambda_L}{\xi} = \sqrt{\frac{2\beta}{\mu_0}}\;\frac{m_s}{\hbar e_s}$$

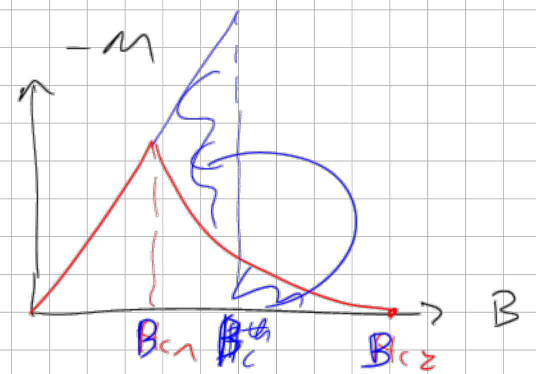
$$\Rightarrow \frac{B_{c2}}{B_c^{th}} = \sqrt{2}\cdot\kappa \quad \Rightarrow B_{c2} = \sqrt{2}\,\kappa\, B_c^{th}$$

6.2.6 Discussion:

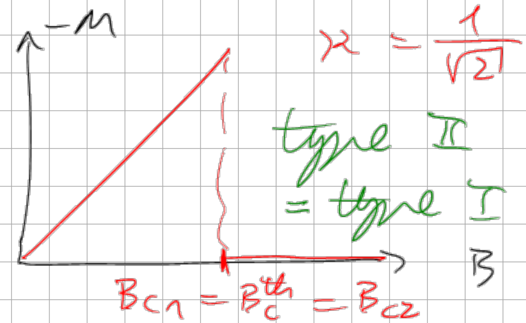
1)

material	T_c	κ
Nb_3Sn	18 K	30
$YBa_2Cu_3O_7$	93 K	100

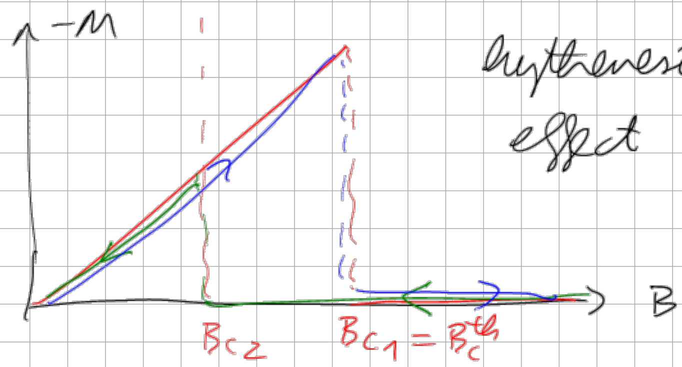
$\Rightarrow B_{c2} \gg B_c^{th}$



2) $\kappa_c = \frac{1}{\sqrt{2}}$



3) What happens for a material of type I SC, which has $\kappa < \kappa_c = \frac{1}{\sqrt{2}}$?



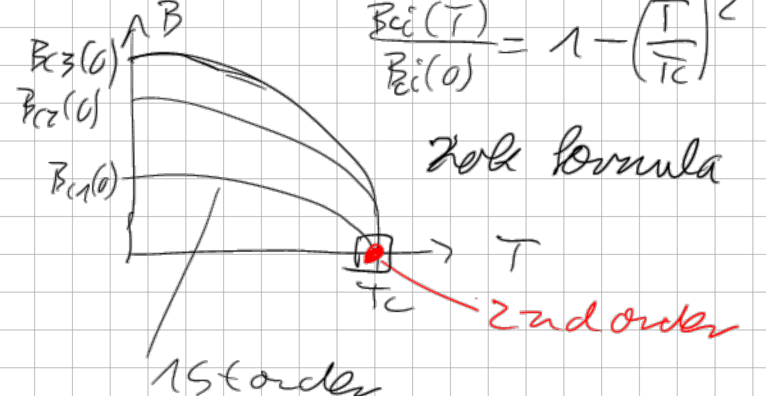
hysteresis effect

aluminium: $\varkappa = 0.03$

pure indium: $\varkappa = 0.06$

Experimental check for $\varkappa_c = \frac{1}{\sqrt{2}}$

4) $B_{c2} \sim (T_c - T)^{\gamma}$



$$\frac{B_c^i(T)}{B_c^i(0)} = 1 - \left(\frac{T}{T_c}\right)^2$$

Kolk formula

Ginzburg-Landau theory $\hat{=}$ mean field theory

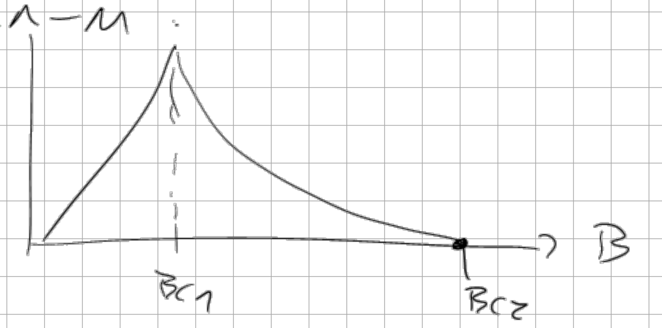
$\Rightarrow \gamma_{MF} = 1$

Due to thermal fluctuations: $\gamma = 0.64$

5) $B_{c2} = \frac{\phi_0}{2\pi\,\xi^2}$, $\phi_0 = \frac{h}{e_s}$

6) $B_c^{th} = \frac{\phi_0}{2\sqrt{2}\pi\,\lambda_L\,\xi}$

NOT fixed



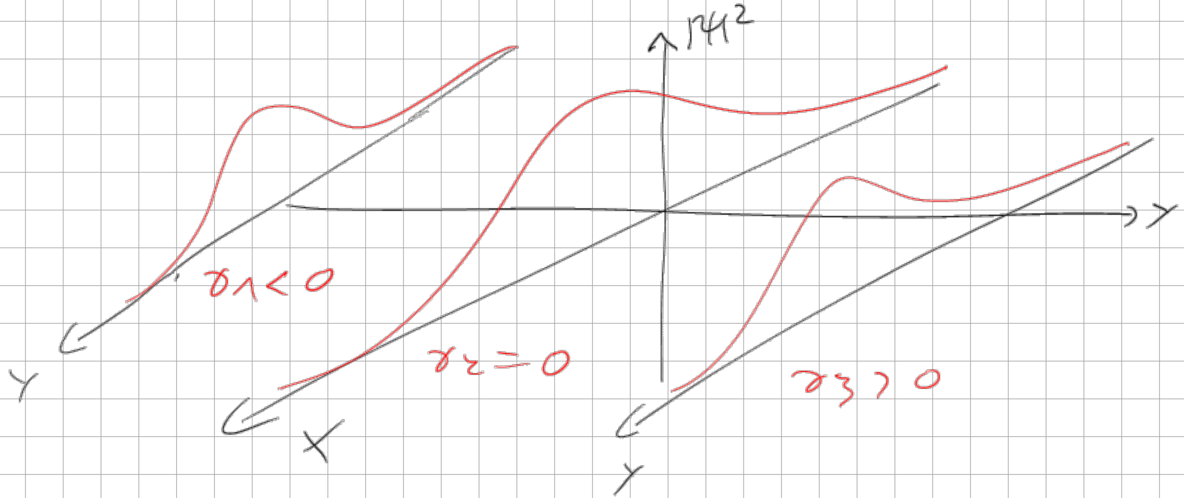
6.2.2 Flux line Lattice:

$n = 0 \Rightarrow \psi_0(x') \sim e^{-\frac{1}{2}\left(\frac{x'}{l_{osc}}\right)^2}$, $x' = x - \frac{\hbar k_y}{e_s B_z}$ (k_y)

oscillator length: $l_{osc} = \sqrt{\frac{\hbar}{m_s \omega_0}}$ $\quad \underline{\underline{\omega_0 = \frac{e_s B_z}{m_s}}}$ $\quad = \sqrt{\frac{\hbar}{e_s B_z}}$

$B \gg B_{c2} \Rightarrow l_{osc}$ is small $\Rightarrow l_{osc} \ll \xi$

$$l_{osc}(B_z) = l_{osc}(B_{c2}) = \sqrt{\frac{\hbar^2}{2 m_s (T_c - T)\, a_c'}} = \xi$$

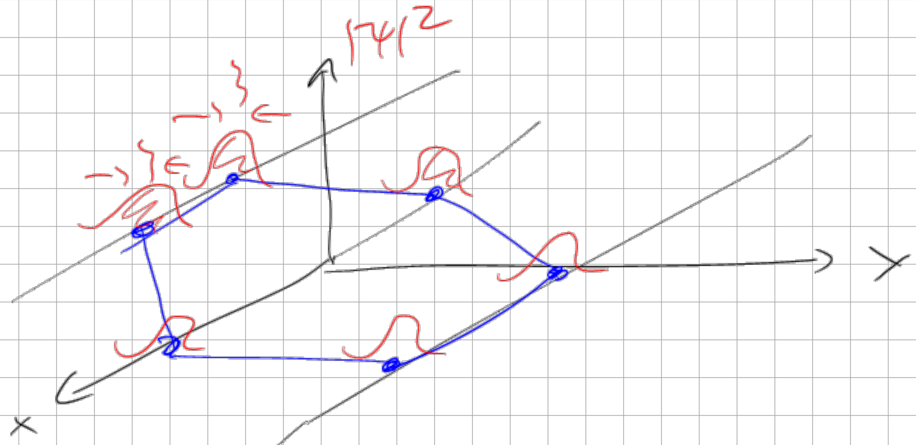


no flux lattice

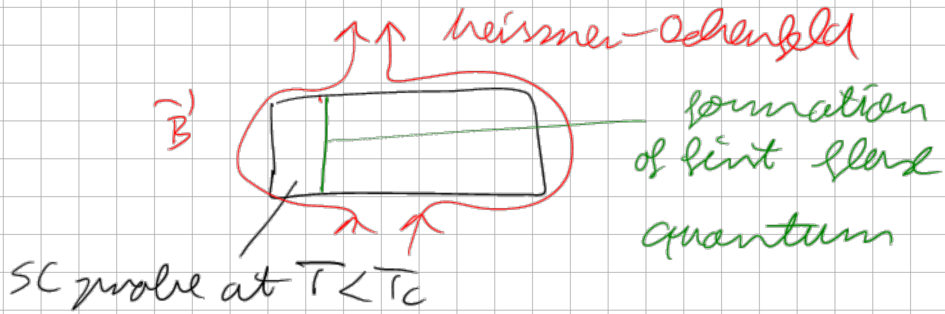
γ is not fixed: $B_z = B_{c2}$

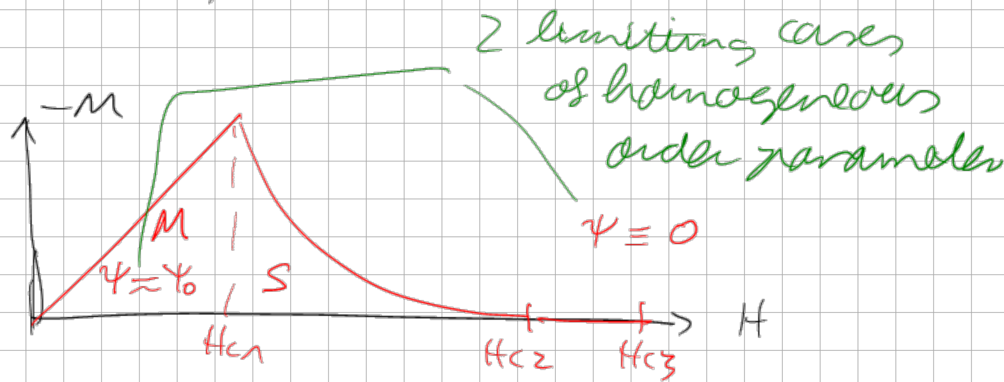
$$\psi(x,y,z) = \int d\gamma \; c(\gamma)\, e^{i\gamma y}\, e^{-\frac{1}{2\xi^2}\left(x - \frac{\hbar \gamma}{e_s B_{c2}}\right)^2}$$

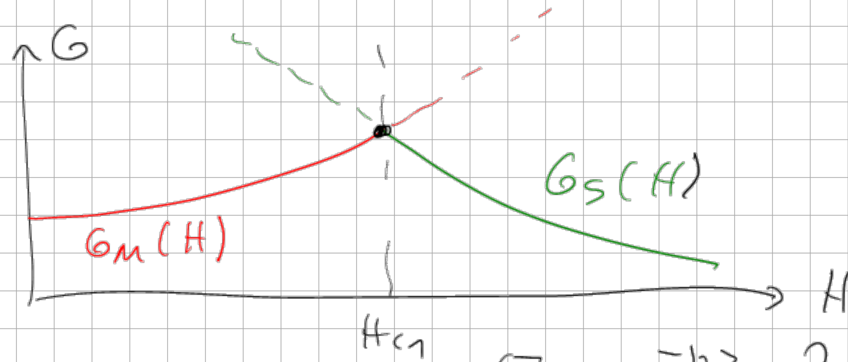
- Abrikosov: quadratic flux lattice
- hexagonal flux lattice
  → experimentally found



6.4 Lower Critical Field H_{c1}:







$$G_M(H_{c1}) = G_S(H_{c1})$$

→ condition for determining H_{c1}

6.3.1 Starting Point

$$G_S = G_n + \int_V dV \left\{ \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m_s}\left|(-i\hbar\vec{\nabla} - e_s\vec{A})\psi\right|^2 + \frac{(\vec{B} - \mu_0\vec{H})^2}{2\mu_0} \right\}$$

assumption: neglect any surface effect

6.3.2 Meissner Phase:

$$\psi = \psi_0 \,, \quad \vec{\nabla}\psi = \vec{\nabla}\psi^* = \vec{0} \,, \quad \vec{B} = \vec{A} = \vec{0}$$

$$G_M = G_n + \int_V dV \left\{ \alpha|\psi_0|^2 + \frac{\beta}{2}|\psi_0|^4 + \frac{\mu_0}{2}\vec{H}^2 \right\}$$

6.3.3 Shubnikov Phase:

H is a little bit above H_{c1}: first flux quantum arises

equilibrium condition:

$$0 = \int_V dV \left\{ \underbrace{\alpha\left[|\psi|^2 - |\psi_0|^2\right] + \frac{\beta}{2}\left[|\psi|^4 - |\psi_0|^4\right] + \frac{1}{2m_s}\left|(-i\hbar\vec{\nabla} - e_s\vec{A})\psi\right|^2 + \frac{\vec{B}^2}{2\mu_0}}_{①} - \underbrace{\vec{B}\cdot\vec{H}_{c1}}_{②} \right\}$$

non-vanishing in vicinity of flux quantum: $V = F\cdot L$, $dV = dF\,L$

interaction of flux quantum with magnetic external field

$$① = L \cdot \underbrace{\varepsilon_L}_{\text{line energy of a single flux quantum}}$$

$$[\varepsilon_L] = 1 \, \frac{J}{m}$$

$$② = -H_{c1} \underbrace{\int_F B \, dF}_{= \phi_0} =$$

$$H_{c1} = \frac{\varepsilon_L}{\phi_0}$$

next time

approximation $\kappa \gg 1$, extreme type SC

$$B_{c1}(\kappa) = \sqrt{2} \, B_c^{th} \, \frac{1}{2\kappa} \left(\ln \kappa + \frac{7}{12} \right)$$

valid for $\kappa > 3$