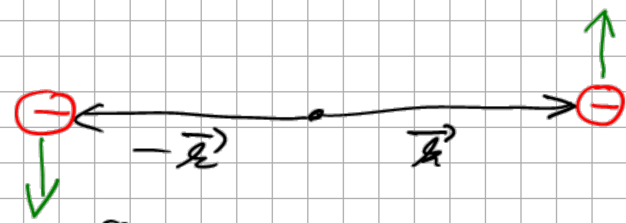
1.4 BCS Theory:



Cooper pairs:

- pairing in Fourier space
- distance between 2 electrons: 1000 Å ⟷ atomic distances of Å
- energetically pairing occurs in a shell around Fermi energy
  → shell of size $\underset{\sim}{\Delta}$: no single electrons, all electrons are paired
  energy gap: low-temperature dependence of heat capacity
- Landau - Khalatnikov two-fluid model: $\underbrace{n}_{\text{density of electrons}} = \underbrace{n_s}_{\text{superconducting electrons}} + n_n$ normal-conducting
  superconducting electrons ≙ electrons bound by cooper pairs
- Consequence of cooper pairing:

$$T_c = 1.13 \underbrace{\frac{\hbar\omega_D}{k_B}}_{T_D =} e^{-\frac{2}{N(E_F)\, g}}$$

g: effective attractive electron-electron interaction

$N(E_F)$: density of states for electrons at Fermi energy

ω_D: Debye frequency, largest phonon frequency

niobium: $T_D \approx 270\,K$

niobium: 70.000 K

T_c: critical temperature

niobium: 9 K

$$e^{-\frac{1}{x}} \overset{x \geq 0}{=} \underbrace{0 + 0 \cdot x + 0\, x^2 + \ldots}_{\text{Taylor-series vanishes}}$$

BCS theory explains a posteriori some observations from last lecture

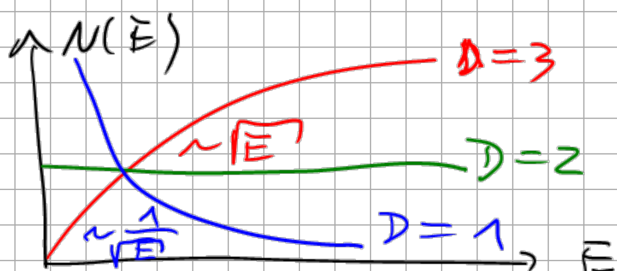
1) Usually electrons are hampered by interactions with phonons:
 - Cooper pair electrons are bound
 - normal conducting electrons one need energy in order of Δ

2) Matthias rule:

odd number of electrons $\rightarrow$ large $N(E_F)$ $\rightarrow$ large T_c

even " " " small $N(E_F)$ $\rightarrow$ small T_c

3) Nb_3Ge $\hat{=}$ 1D electrons $\Rightarrow$ largest T_c for conventional SC



And: $T_D \sim \frac{1}{\sqrt{M}}$ $\Rightarrow$ $T_C \sim \frac{1}{\sqrt{M}}$ isotope effect $\Rightarrow \sqrt{M} \cdot T_C \approx$ const.

- measured in 1950
- high T_c SC do NOT fulfill the isotope effect

$\Rightarrow$ chapter 2: magnetic properties

$\Rightarrow$ Chapter 3: London theory

2 Magnetic Properties:

superconductor = ideal conductor + ideal diamagnet

2.1 Meissner-Ochsenfeld Effect: (Berlin, 1933)

ideal conductor: Maxwell equations

1) Ohm law (matter equation)

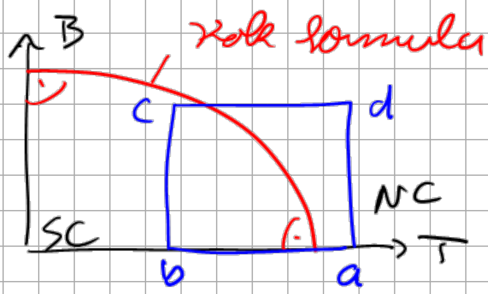
$$\vec{j} = \underbrace{\sigma}_{\text{conductivity}} \vec{E} \quad \Rightarrow \quad \lim_{\rho \downarrow 0} \rho \vec{j} = \lim_{\rho \downarrow 0} \vec{E} = \vec{0}$$

$$= \frac{1}{\rho} \leftarrow \text{resistivity}$$

2) Induction law (Maxwell equation)

$$\mathrm{rot}\, \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \underset{\rho \downarrow 0}{\Longrightarrow} \quad \lim_{\rho \downarrow 0} \frac{\partial \vec{B}}{\partial t} = \vec{0} \Rightarrow \vec{B}(t) \equiv \text{const. in time}$$

Meissner-Ochsenfeld: $\vec{B} = \vec{0}$

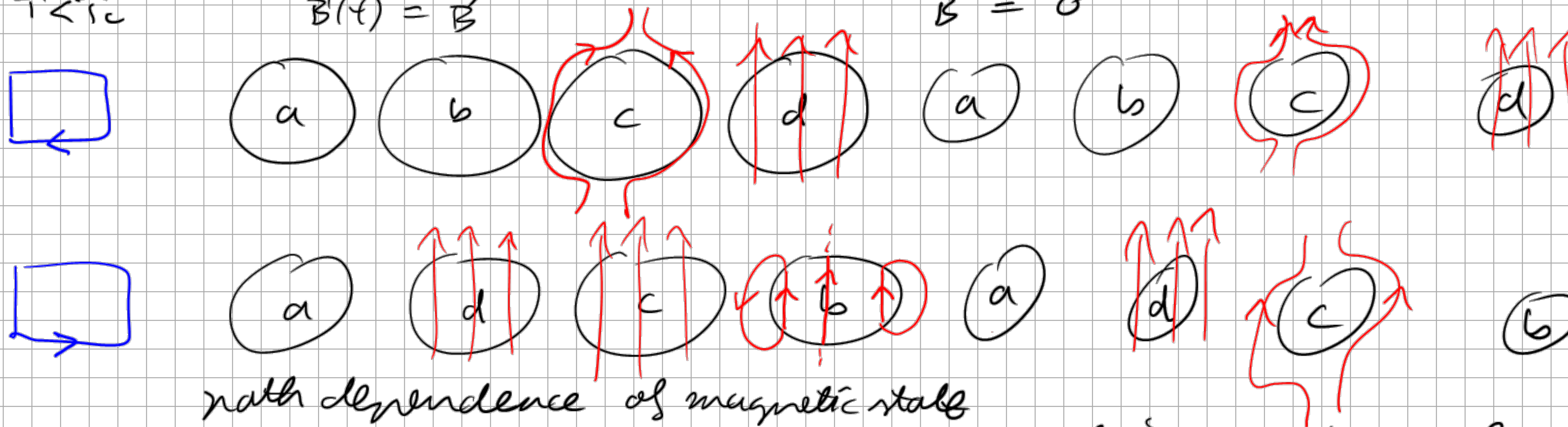


cycle process

side remark: sphere

simplification: homogeneous magnetic field ideal conductor $\qquad\qquad\qquad\qquad$ superconductor

$T<T_c$ $\qquad$ $\vec{B}(t)=\vec{B}$ $\qquad\qquad\qquad$ $\vec{B}=\vec{0}$



path dependence of magnetic state

path independence of magnetic state

Two important conclusions:

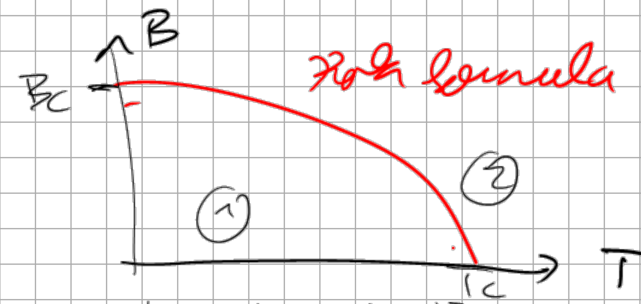
1) Path independence implies thermodynamic description
   → chapter 3: Thermodynamic Properties

2) Electrodynamic theory has to be modified for superconductors
   → Chapter 4: London theory

Microscopic explanation for Meissner-Ochsenfeld effect:

- magnetic induction decreases on London length scale of order of 100 Å
- superconducting currents in thin skin which compensate external magnetic induction

2.2 Type I Superconductor:



$$\frac{B(T)}{B_c} = 1 - \left(\frac{T}{T_c}\right)^2$$

$$B(T) = \mu_0 H(T)$$

① SC phase: shows in volume Meissner-Ochsenfeld effect

② normal conducting phase

Note: concentrate on sphere

$$\underset{\text{magnetic induction}}{\vec{B}} = \vec{B}_{ext} + \vec{B}_{int}$$

magnetic induction (moving charged particles feel a force)

$$\vec{B}_{ext} = \mu_0 \underbrace{\vec{H}}_{\text{magnetic field}}$$

magnetic field (generated by external currents)

$$\underbrace{\vec{B}_{int}}_{\text{infinitely large body}} = \mu_0 \underbrace{\vec{M}}_{\text{magnetization}}$$

magnetization (stems from internal currents)

$$\vec{M} = \chi_m \vec{H}$$

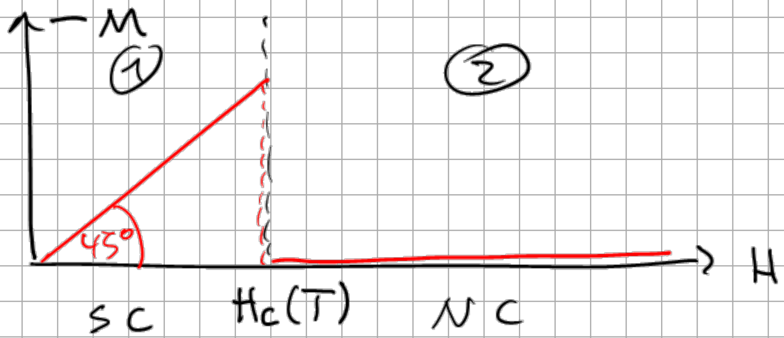
magnetic susceptibility

$$\Rightarrow \quad \vec{B} = \mu_0 (1 + \chi_m) \vec{H} \overset{!}{=} 0$$ Meissner-Ochsenfeld effect

$$\Rightarrow \quad \chi_m = -1 \quad (\text{"ideal diamagnet"})$$

normal conductor: $\chi_m \gtrsim 0 \Rightarrow \chi_m = 0$

magnetization curves: two representations



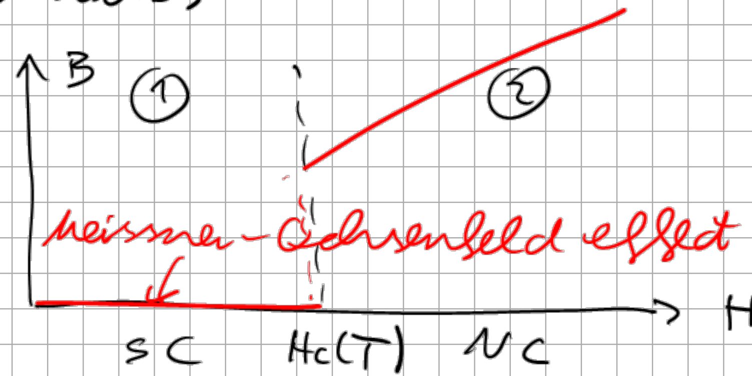


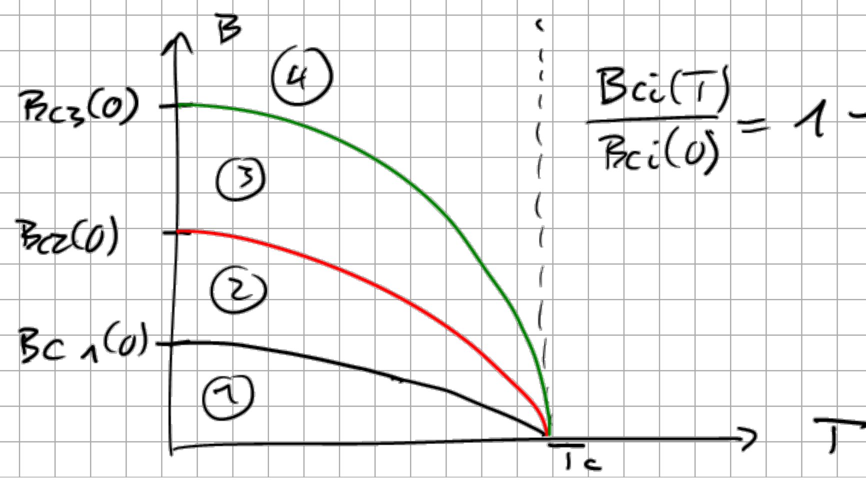
Table: $B_c(0) = \mu_0 H_c(0)$ in T

Pb	0.0803	small ≙ "soft" superconductor
Ta	0.0830	
Al	0.0990	
V	0.131	

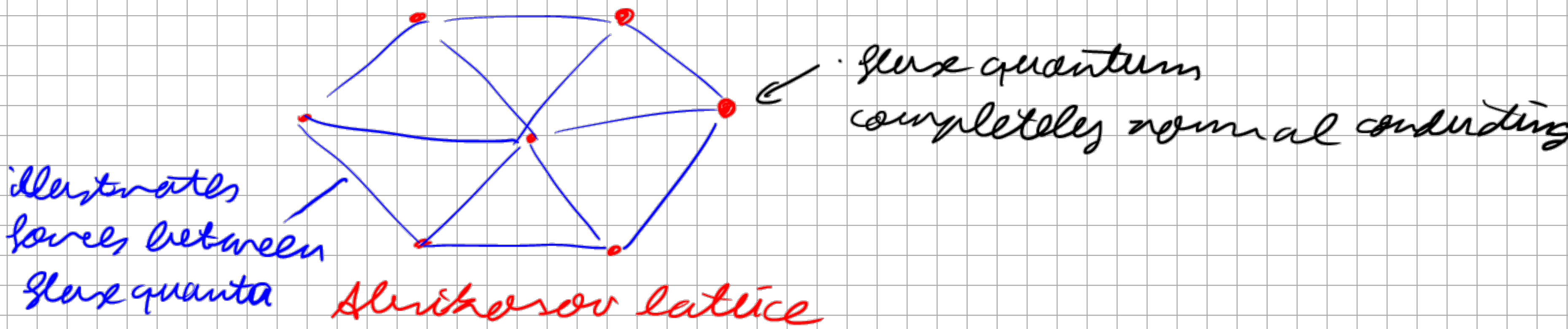
2.3 Type II superconductors:

① SC: Meissner-Ochsenfeld phase

② Shubnikov phase



$$\frac{B_{ci}(T)}{B_{ci}(0)} = 1 - \left(\frac{T}{T_c}\right)^2$$



(3) surface phase: SC exists only at border of superconductor

(4) normal conducting phase

<u>magnetic properties of type II SC</u>

(1) Meissner phase: Ochsenfeld effect $\quad 0 \le B \le B_{c1}(T)$

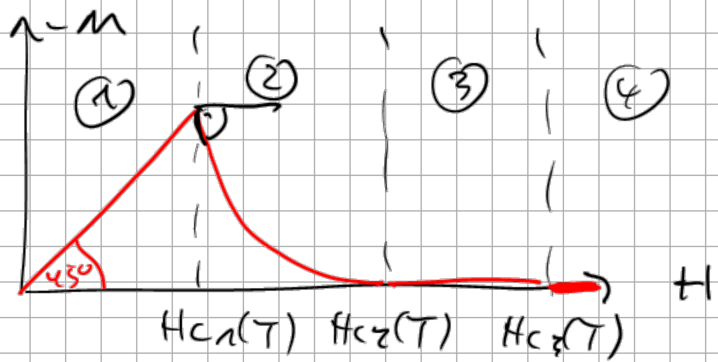
(2) Shubnikov phase $\quad B_{c1}(T) \le B \le B_{c2}(T)$

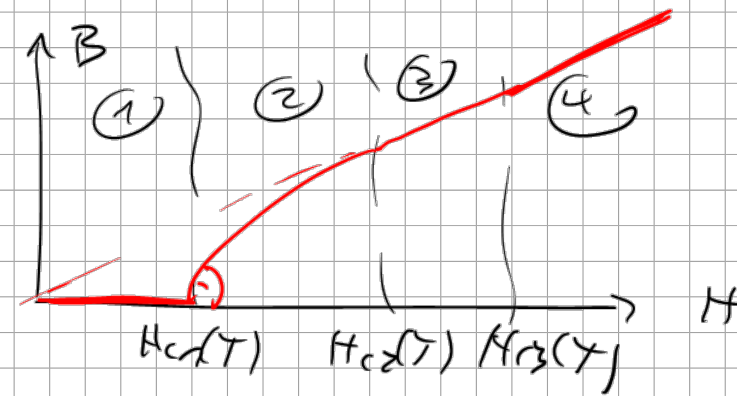
more and more NC regions with $\chi_m = 0$

(3) Surface phase: $\quad B_{c2}(T) \le B \le B_{c3}(T)$

$M = 0$ in volume

(4) NC: $\quad B \ge B_{c3}(T) \quad \rightarrow \chi_m = 0$





	Nb	NbTi	Nb_3Sn	$YBa_2Cu_3O_7$
$B_{c2}(0)$ in T	0.1944	14	25	$\sim 10^2$

$\Rightarrow$ "hard" SCs

Next lecture: Tuesday, 15.45, 46-576